Math Monstrosity Tiebreakers

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.

If a question begins with "paper and pencil ready", it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.

If, at any time during an equation, you see something like $\frac{\textbf{THIS}}{2}$ or $\textbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as "this function" or "this quantity" or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say "this thing" or "this".

Pronounciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.

Please read Greek letters as they are (for example, read ϕ as "phi" and not "the golden ratio", even if it represents the golden ratio), with the notable exception of $\sum$ and $\prod$, as in $\sum_{n=1}^{5}$, which should be read as "the sum from $n = 1$ to 5 of".

Similarly, $\int_a^b$ is "the integral from a to b" and $\lim_{n\to\infty}$ is "the limit as n approaches infinity".

In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as "f of x" and "lambda of u and v" respectively, and not as "f x" and "lambda u v".

Please read large and/or complex fractions by saying "in the numerator", reading the numerator, saying "in the denominator", reading the denominator, and then saying "end of fraction". For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read "a over b squared plus c".

Please read $\binom{a}{b}$ as "a choose b", not as "a over b".

If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means "the transpose of A", read it as "A to the power of T" or "A superscript T". Most of the notation used in this tournament is common enough that such descriptions, using words like "subscript" and "superscript", should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. A closed one of these is called essential if it has nonzero intersection with every other nonzero closed one of these things. If these things are maximal, then they are prime, meaning the set of elements not in them is closed under multiplication. Operations on these include ● contraction and expansion, which consist of putting them through a homomorphism or through its inverse, respectively. One of these things is called principal if it is generated by a single element. For ten points, identify these subsets of rings which consist of additive groups closed under multiplication.

Answer: **ideal**s

2. The non-associative system of Cracovian multiplication was introduced to help manually compute solutions to these things. The Moore-Penrose pseudoinverse can be used to solve these things, or approximate solutions if none exist. Levinson recursion can be used to solve examples of these things if they can be expressed using a Toeplitz ● matrix. These things can be solved using Gaussian elimination on augmented matrices or by using Cramer's rule, and these things can either have one, none, or infinitely many solutions. For ten points, identify these sets of statements which describe how sums of variables are equal to constants.

Answer: **system**s of **linear equation**s [accept things in different order, prompt on equations, prompt on linear equations, prompt on systems of equations, prompt on matrices and word forms]

3. A suspension is defined by the join of one of these with a pair of points. If one of these has a property named for Baire, then each of its subsets of the first category must have an empty interior. These things were defined by Hausdorff using several axioms which are no longer considered necessary, such as the requirement that any two elements of these things be elements of ● disjoint open sets; that is the criterion for these things to be Hausdorff. These things must be closed under arbitrary union and finite intersection. For ten points, identify these mathematical objects which define a collection of open sets.

Answer: **topologies** [accept word forms, accept abstract **topological space**s, prompt on spaces]

4. The "polynomial" one of these things generalizes certain complexity classes to oracle machines. They're not degrees, but one of these things named for Wadge characterizes the complexity of sets of reals. One of these things named for Borel stratifies a Borel algebra, and the "arithmetical" one of these things classifies sets based on the complexity of the formulas used to define them. These things are said to collapse if they have only finitely many elements, which are also called ● "levels". For ten points, identify these objects in set theory which consist of a preorder defined on a set.

Answer: **hierarchies** [accept word forms]

5. A Mesoamerican version of this device was known as the nepohualtzintzin and was used by people known as temalpouhqueh. Skilled operators of these devices can perform a technique known as *anzan*, where they visualize these devices in order to perform mental calculation without using them. The use of these computing devices in Rome originated the use of the word "calculus", originally the word for ● pebble, to refer to computation. The suanpan and the soroban are examples of, for ten points, what sort of arithmetical computing aid which makes use of beads on wires?

Answer: **abacus** [accept word forms, accept **suanpan** and **soroban** before mention, prompt on descriptive answers]

6. Ira Poll has proposed a method for breaking ties in a heuristic algorithm for finding these; that algorithm is Warnsdorf's rule. George Koltanowski was famous for giving performances in which he performed one of these things while blindfolded. One of these things is used to set the order of the chapters in the Oulipo novel *Life a User's Manual*. A Sanskrit poetic work which can be read either normally or by following one of these things is entitled "Arrangement in the steps of a ● horse". Unlike the general Hamiltonian path problem, these things can be found in linear time. For ten points, identify these sequences of chess moves in which a certain piece visits every square on the board exactly once.

Answer: **knight's tour**s [prompt on Hamiltonian paths, prompt on paths, prompt on tours]

7. A "range" described by this adjective is a set of four points A, B, C, and D which are arranged on a line such that $\frac{AC}{CB} = -\frac{AD}{DB}$, where distance is signed. Functions which satisfy Laplace's equation are described using this term, while the branch of analysis which is described by this term concerns the study of ● Fourier transforms. A mean described by this adjective consists of the reciprocal of the sum of the reciprocals of the inputs. For ten points, identify this adjective, which can describe oscillation that has simple and quantum types.

Answer: **harmonic**

8. This mathematician conducted a study in which he found that financial markets' price fluctuations were governed by Lévy stable distributions. This man declared "Clouds are not spheres, mountains are not cones, coastlines are not circles" in a book explaining his theory of "roughness". This man's paper "How Long is the ● Coast of Britain? Statistical Self-Similarity and Fractional Dimension" discusses self-similar curves, and in 1975 this man coined the term "fractal" to describe such shapes. For ten points, identify this mathematician who names a set of complex numbers with fractal properties.

Answer: Benoit B. **Mandelbrot**

9. A form of this property named for Melnikov generalizes it to measures, and the scalar form of this quantity is equal to the product of the metric tensor and the Ricci tensor. One form of this quantity is defined as the determinant of the

shape operator and is named for Gauss. The Frenet formulas express a curve in terms of torsion, starting point, and this quantity, which is equal to the ● magnitude of the rate of change of the tangent vector. This quantity is defined as the reciprocal of its namesake radius. For 10 points, identify this quantity usually denoted kappa which describes how much a surface differs from being flat.

Answer: **curvature**

10. The Plücker matrix, which has this property, characterizes a straight line in projective space. The Pfaffian of a matrix is only nonvanishing if the matrix has this property, and the square of the Pfaffian of a matrix with this property is the determinant. Matrices with this property and odd dimension must be noninvertible by Jacobi's theorem. Any matrix can be written as the sum of a ● symmetric matrix and a matrix with this property, and matrices with this property must have zero on the diagonal. For 10 points, identify this property held by matrices whose negative is their transpose.

Answer: **skew-symmetric** [accept **antisymmetric**, do not accept or prompt on "symmetric"]

11. The Eilenburg-Mazur swindle involves the misapplication of one form of this process which takes a limit of arithmetic means. One formula for doing this is co-named for Plana and is related to one named for Lambert. Watson's theorem and Carleman's theorem guarantee the optimality of one method of doing this which has exponential and integral types and is named for Borel. It's not integration, but a method of doing this "by ● parts" is often also named for Abel. A notational convention in physics for writing this process is named for Einstein. Cesaro and Euler name types of, for ten points, what sort of process which involves finding the limit of a series?

Answer: **summation** [accept descriptive answers, prompt on taking <u>limits</u>]

12. Korte and Lovasz introduced a variety of matroids named after this concept for which this concept is always optimal for them. Kruskal's algorithm is one of these things for the cycle matroid, and in general, these things, along with independence oracles, can be used to determine a maximum-weight basis for matroids. For every number of cities in the traveling salesman problem, one can always assign distances between them such that this type of process finds the unique worst possible solution. In the case of the traveling salesman problem, the ● nearest neighbor algorithm is one of these things. For ten points, identify this general class of algorithm which always makes the best short-term choice.

Answer: **greedy algorithm**s