Math Monstrosity, Packet 9

Conor Thompson
University of Michigan

June 5, 2017

1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.

If a question begins with "paper and pencil ready", it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.

If, at any time during an equation, you see something like $\frac{\textbf{THIS}}{2}$ or $\textbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as "this function" or "this quantity" or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say "this thing" or "this".

Pronounciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.

Please read Greek letters as they are (for example, read ϕ as "phi" and not "the golden ratio", even if it represents the golden ratio), with the notable exception of $\sum$ and $\prod$, as in $\sum_{n=1}^{5}$, which should be read as "the sum from $n = 1$ to 5 of".

Similarly, $\int_a^b$ is "the integral from a to b" and $\lim_{n \to \infty}$ is "the limit as n approaches infinity".

In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as "f of x" and "lambda of u and v" respectively, and not as "f x" and "lambda u v".

Please read large and/or complex fractions by saying "in the numerator", reading the numerator, saying "in the denominator", reading the denominator, and then saying "end of fraction". For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read "a over b squared plus c".

Please read $\binom{a}{b}$ as "a choose b", not as "a over b".

If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means "the transpose of A", read it as "A to the power of T" or "A superscript T". Most of the notation used in this tournament is common enough that such descriptions, using words like "subscript" and "superscript", should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. Nevanlinna and Pick name a problem of performing this process on the complex unit disk, and Abel and Goncharov name a type of this process which matches higher derivatives. Newton names a polynomial used for this process the coefficients of which are calculated using divided differences. ● Runge's phenomenon occurs when this process becomes less accurate as the degree of the polynomials used in it increases, and can be avoided by using unequal intervals. The simplest method of doing this, the linear method, connects consecutive data points with lines. For ten points, identify this process from numerical analysis in which a data set is "filled in" by a function.

Answer: **interpolation** [prompt on approximation, be nice and prompt on extrapolation]

2. If a patch is regular and has this property, then it maps its domain diffeomorphically onto its codomain. Every cofree module has this property, and a module M is said to have this property if it is a direct summand of any module N of which it is a submodule. A functor having this property on maps is called faithful, and if it has this property on objects as well, it is called an ● embedding. Continuous functions from $\mathbb{R}$ to $\mathbb{R}$ with this property must be monotone. If there exists a function with this property from X to Y and also from Y to X, then X and Y must have the same cardinality. For ten points, identify this property held by a function f if $f(x)$ and $f(y)$ are always distinct elements of the codomain.

Answer: **injectivity** [accept word forms]

3. One group of three mathematicians from this country is known as the Three Arithmeticians, and Soddy's hexlet was independently discovered in this country in 1822. Another mathematician from this country used a system called "circle principles" to calculate limits like the area of a circle. The oldest surviving mathematical text from this country concerned computation techniques using an ● abacus and was titled the Jinkōki. This country's practice of leaving geometric problems on wooden tablets as offerings at temples is known as sangaku. A mathematician from this country developed inter-universal Teichmüller theory while working at Kyoto University. For ten points, identify this country of Shinichi Mochizuki.

Answer: **Japan** [accept **Nippon**-koku, accept **Nihon**-koku]

4. The polar version of this function is applied to n-dimensional parallelotopes and simplices. The integral from zero to infinity of this function of x^n is equal to $\Gamma\left(1 + \frac{1}{n}\right) +$ **THIS** $\left(\frac{\pi}{2n}\right)$. The integral of the inverse of this function of x is equal to $\sqrt{1 - x^2} + x \cdot$ **THIS**$^{-1}(x) + C$. The ● hyperbolic version of this function is equal to $\frac{e^x - e^{-x}}{2}$. This function of x times i is the imaginary part of e^{ix}, and this function's second derivative is equal to its negative. For ten points, identify this trigonometric function equal to the reciprocal of cosecant.

Answer: **sine** [accept "cosine" until the gamma function is read]

5. The Chakravala method is used for finding solutions to these things. The Erdos-Strauss conjecture states that a certain set of these things all have solutions. Hilbert's tenth problem asked whether all of these things could be solved, and ● Matiyasevich proved that no general algorithm for solving these things existed. Pell's equation is one of these things, which can be solved using the Hasse principle. Fermat's Last Theorem states that no solutions exist to a specific class of these equations. For ten points, identify these equations to which only the integer solutions are relevant to study, named for a classical mathematician from Alexandria.

Answer: **Diophantine** equations

6. A set S shatters A if this thing for A is equal to the set of intersections of elements of S with A. The existence of one of these for every set is one of the axioms of Zermelo-Fraenkel set theory. Cantor's paradox is that the set of all sets is its own one of these, and therefore must be bigger than itself. The singleton function, which takes a set X to this thing of X, maps every element of X to the singleton set containing it. If a set has cardinality n, then this thing for it has cardinality ● 2^n. For ten points, identify this object associated with a set X which has as elements all of the subsets of X.

Answer: **power set** [prompt on descriptive answers like "set of subsets of a certain set"]

7. Hodge et al. showed that this property's namesake coefficient is equal to $\frac{1}{A_D}$ times the double integral over D of $A_D(x,y)$. A set is called a cone with this property if for all x, y in the set and scalars a and b it's the case that $ax + by$ is also in the set. Every simple polygon has at least one mouth unless it has this property. A set is ● star-this if there exists a point P such that the line segment XP is contained in the set for every point X in the set. The this hull of a set is the intersection of all sets with this property containing it. For ten points, identify this property which is held by polygons with no exterior diagonals.

Answer: **convexity** [accept word forms]

8. A team from Southampton University made 60 entries into a competition for this game, which colluded with one another to ensure that a few of them would sweep the top spots. Robert Axelrod's book The Evolution of Cooperation describes a number of different strategies for this game, and concludes that they must be nice, retaliating, forgiving, and non-envious. William H. Press and Freeman Dyson developed the theory of ● zero-determinant strategies for the stochastic version of this game. Win-stay lose-switch, also known as Pavlov, and Anatol Rapaport's tit-for-tat are two strategies for, for ten points, what game in which a certain scenario about defecting or cooperating is played out repeatedly?

Answer: **iterated prisoner's dilemma** [prompt on prisoner's dilemma, accept answers that consist of repeatedly saying the words "prisoner's dilemma"]

9. In tensor fields, this operation of A superscript alpha is defined as epsilon superscript alpha mu nu times A subscript nu semicolon mu. In Maxwell's

equations, this operation of B is equal to $\mu_0 \mathbf{J} + \epsilon_0 \mu_0$ times the partial derivative of $\mathbf{E}$ with respect to t. A vector field u such that u cross this operation of u is zero is said to be a ● Beltrami field. This operation's namesake theorem states that the integral over S of this function of F da is equal to the integral over partial A of F ds. This operation is equal to the cross product of the gradient vector with a vector field. A vector field where this quantity is zero at all points is known as irrotational. For ten points, identify this operation, alternatively known as "rotor" and representing the circulation of a vector field around a point.

Answer: **curl** [accept **rotor** or **rotational** before read, accept **del cross** before "cross product" and prompt afterward]

10. This man classified 72 of the 78 species of cubic curves, and names a set of identities relating power sums to symmetric polynomials. This mathematician received special permission not to take holy orders upon receiving a professorship. He names a theory of color stating that objects' color comes from their interaction with light. He was appointed Master of the ● Mint and successfully prosecuted counterfeiters, resigning his post at Cambridge to concentrate on that work. This man published a work in which he advanced a corpuscular theory of light, and he also discovered the generalised binomial theorem. For ten points, identify this English mathematician and physicist who names three laws of motion.

Answer: Sir Isaac **Newton**

11. Bauer's theorem discusses the divisors of a polynomial described by this term. A cuboid whose edges and face diagonals are members of this set is called an Euler brick. Gauss's theorem says that a polynomial described by this term can be factored into two polynomials with rational coefficients iff it can be factored into two polynomials with these coefficients, and an algebraic ● congruence is an equation of the form $f(x) \equiv 0 \bmod n$ where $f(x)$ is a polynomial described by this term. Complex analogues of this set include the Eisenstein and Gaussian types of these. For ten points, identify this set of numbers represented by a double-struck Z.

Answer: **integer**s [prompt on counting numbers, prompt on natural numbers]

12. Sermanet and LeCun used multi-scale convolutional networks to recognize these objects, and a 2003 article in Image and Vision Computing by de la Escalera develops an early neural network to recognize these objects. Dan Ciresan developed a neural network that won a 2011 prize for recognizing these objects. The Swedish business Mapillary is facilitating the computer recognition of these objects by building a crowdsourced alternative to Google ● Street View. These objects could be replaced with a system called IVS, and their recognition is important to the operation of self-driving cars. For ten points, identify these objects which say things like "New York City 30 Miles" and "Right Lane Exit Only" to drivers.

Answer: **traffic sign**s [accept any synonymous answers indicating that they're signs found on roads, prompt on <u>signs</u>]

13. The Nielsen-Schreier theorem states that any subgroup of one of these groups is another one of these groups, and the instance of this group on two elements is SQ universal. Any group is the homomorphic image of one of these, and these groups are not abelian unless their generating set has only one element. The fundamental group of the figure eight is an example of one of these groups, and the group of integers under addition is isomorphic to this with one generator. These groups are made up of ● words, and the operation on those words is concatenation. For ten points, identify this class of groups where there is no relationship between group generators other than between an element and its inverse.

Answer: **free** groups

14. The first engineered one of these things made use of the 17c/45 reaction in its construction. One of these was discovered in 2016 with speed c/10 and is known as the copperhead. Flotillae of these things often involve overweight ones which cannot survive otherwise. The four smallest examples of these things include their lightweight, middleweight, and heavyweight types, and if one of these things leaves debris behind, it is known as a puffer. These things can be emitted by ● guns, which include a famous example named for Gosper. The glider is the most common example of, for ten points, what objects in Conway's Game of Life which move by returning to their original state but in a different location?

Answer: **spaceship**s [antiprompt on <u>gliders</u> and <u>knightships</u>]

15. This function is defined in general by a sum of elements of the form $\frac{\sin(\pi k^a x)}{\pi k^a}$. This function's graph has a Hausdorff dimension of $2 + \frac{\ln a}{\ln b}$, where a and b are parameters used to define it, $0 < a < 1$, and b is a positive odd integer greater than or equal to 7. This function is Hölder continuous but not ● Lipschitz continuous at all points, and is considered one of the monsters of real analysis. This function is only differentiable at a rational number if it is expressible as the ratio of two odd numbers. For ten points, identify this pathological function which is continuous everywhere but differentiable on a set with Lebesgue measure zero.

Answer: **Weierstrass function**

16. The Gömböc is a shape whose stable and unstable points of equilibrium have this property. A graph is said to be chromatically this if it is not chromatically equivalent to any other graph to which it is not isomorphic. Solutions to Langford's problem modulo 3 and 4 have this property up to digit reversal. A prime number p is said to have this property if $\frac{1}{p}$ is a repeating decimal with period n, and for no ● other prime number q does $\frac{1}{q}$ have period n. Up to rotations and reflections, a solution to the six queens puzzle has this property.

For ten points, identify this property of being the only member of a certain set or solution to a certain problem.

Answer: **unique**ness [accept word forms]

17. The faltung of two bilinear forms has this property if both of those bilinear forms has this property. A lattice is said to have this property if it has elements called 1 and 0 which fulfill certain properties. Any holomorphic function with this property must be a ● constant function, by Liouville's theorem. Every sequence with this property must have a convergent subsequence by the Bolzano-Weierstrass theorem, which also implies that compact sets in $\mathbb{R}^n$ must be closed and have this property. A set has this property if it is contained in a ball of finite diameter. Suprema and infima are possessed by sets with, for ten points, what property which is held by intervals with two finite endpoints?

Answer: **bounded**ness

18. The Gershgorin Circle Theorem guarantees that each of these will be contained in a certain set of disks. The Lanczos algorithm is used to compute these non-vector quantities, and Lyapunov's First Theorem gives necessary and sufficient conditions to ensure that the real parts of these quantities will be negative. These values are invariant under similarity transformations, and Schur's inequalities relate these values to the entries of the ● matrix from which they are calculated. These values are also known as characteristic roots because they are the roots of a matrix's characteristic polynomial. For ten points, identify these scalars associated with a matrix and corresponding to vectors with the same prefix.

Answer: **eigenvalue**s [accept **characteristic root**s before read, accept **characteristic value**s, accept **proper value**s, accept **latent root**s]

19. *The Reaction in Germany* attributes this statement to the Juste-Milieu among similar statements attributed to the Left and the Right, and the narrator of *Notes from Underground* describes this statement as "sometimes a very charming thing too". Eugene Lyons's *Assignment in Utopia* includes a chapter titled after this statement. Victor Hugo lamented popular support for Napoleon III by declaring "Now, cause seven million five hundred thousand voters to declare that [this statement is true]". In another novel, the narrator declares "the ● Party would announce [this], and you would have to believe it"; that is George Orwell's *1984*. For ten points, identify this false statement about the addition of even primes.

Answer: **two** plus **two** is **five** [accept alternate phrasings like "two and two makes five" as long as it is clear that two is being added to itself and the result is five; accept "two times two is five" and similar alternate phrasings as well]

20. In French, these mathematical objects are known as corps, meaning "body". Adjoining a primitive root of unity to the rationals gives the cyclotomic variety of these objects. Any of the number type of these objects is an extension of the rationals. The ● quadratic variety of these objects are composed of

numbers of the form $a+b\sqrt{D}$, where D is squarefree, and examples of quadratic these include the one which contains the Eisenstein integers and the one which contains the Gaussian integers. Finite examples of these constructs with prime order p correspond to the residue classes mod p, and these objects must have zero and one elements. For ten points, identify these algebraic structures which include the real and complex numbers.

Answer: __field__s