Math Monstrosity, Packet 8

Conor Thompson
University of Michigan

June 5, 2017

1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.
If a question begins with "paper and pencil ready", it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.
If, at any time during an equation, you see something like $\frac{\textbf{THIS}}{2}$ or $\textbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as "this function" or "this quantity" or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say "this thing" or "this".
Pronounciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.
Please read Greek letters as they are (for example, read ϕ as "phi" and not "the golden ratio", even if it represents the golden ratio), with the notable exception of $\sum$ and $\prod$, as in $\sum_{n=1}^{5}$, which should be read as "the sum from $n = 1$ to 5 of". Similarly, $\int_a^b$ is "the integral from a to b" and $\lim_{n\to\infty}$ is "the limit as n approaches infinity".
In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as "$f$of x" and "lambda of u and v" respectively, and not as "f x" and "lambda u v".
Please read large and/or complex fractions by saying "in the numerator", reading the numerator, saying "in the denominator", reading the denominator, and then saying "end of fraction". For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read "a over b squared plus c".
Please read $\binom{a}{b}$ as "a choose b", not as "a over b".
If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means "the transpose of A", read it as "A to the power of T" or "A superscript T". Most of the notation used in this tournament is common enough that such descriptions, using words like "subscript" and "superscript", should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. If a Lie group has this property, every representation of it is equivalent to a unitary representation; Lie groups have this property if their parameters vary over a closed interval. The space $\mathbb{R}^n$ can be endowed with this property by adding a point at ● infinity. The Hilbert cube has this property by Tychonoff's theorem, which states that this property is productive, that is, the product of any number of objects with this property also has this property. Any finite space has this property. If a space is Hausdorff, then any set in that space with this property is closed. For ten points, identify this property held by a set if every covering of it with open sets has a finite subcover.

Answer: **compact**ness

2. This non-Weil mathematician names a conjecture that was proved in 1973 by Pierre Deligne, who won a Fields Medal for that work. This mathematician co-developed a series which can be used to exactly compute the number of ● partitions of an integer. This man notably kept his results without proof in looseleaf notebooks, confounding future generations of mathematicians. This man collaborated extensively with Hardy, and the two name the smallest number expressible as the sum of two cubes in two different ways. For ten points, identify this Indian mathematician and inventor of mock theta functions.

Answer: Srinivasa **Ramanujan** Iyengar

3. This man's doctoral thesis was entitled "Discrete groups, analytic groups and Poincaré series". This man published an article in Seed magazine entitled "Prime Numbers Get Hitched" which related the number 42 to the Riemann zeta function. In one of this man's books, he recounts a trip to the Alhambra palace in which he attempts to find examples of every symmetry group; the book also describes the quest to categorize all the finite simple groups and was entitled *Finding ● Moonshine* in the UK and *Symmetry: A Journey into the Patterns of Nature* in the US. This man's first book tells the history of prime number mathematics and the Riemann hypothesis. For ten points, identify this Oxford university professor and author of *The Music of the Primes*.

Answer: Marcus **du Sautoy**

4. The Hurewicz theorem and Blakers-Massey theorem are used to calculate this type of group; the latter process is also known as excision for these groups. Topological spaces with different kinds of this action's namesake group can never be homeomorphic, but the converse of that statement is false. This kind of action preserves ● path-connectedness and the structure of fundamendal groups. Two spaces are isotopic if every intermediate step of one of these actions between them is an embedding. There does not exist one of these between the torus and the sphere, because they are topologically distinct. For ten points, identify this mathematical term for a continuous deformation of a space.

Answer: **homotopy** [prompt on continuous deformation]

5. One mathematician with this surname married a granddaughter of Leonhard Euler before drowning in the Neva river. Another mathematician with this surname posed the problem of reciprocal orthogonal trajectories, while another introduced the St. Petersburg paradox, which was resolved by yet another mathematician with this surname. The differential equation $y' + P(x)y = Q(x)y^n$ is named for a mathematician with this surname, and another mathematician with this surname discovered the rule that was eventually named for ● l'Hôpital. Two mathematicians with this surname solved the brachistochrone problem. For ten points, identify this surname held by a family of Swiss mathematicians.

Answer: **Bernoulli**

6. This shape can be described in Cartesian coordinates by the equation $\left(c - \sqrt{x^2 + y^2}\right)^2 + z^2 = a^2$. This shape names a class of knots including the trefoil knot and the Solomon's seal knot. This shape comes in ring, spindle, and horn varieties, and ● Villarceau circles are located on this shape. This shape can be constructed from a rectangle by gluing opposite sides together without twists, and consistent with the Heawood conjecture, it requires seven colors to color every map on it. For ten points, identify this shape which has a hole in it, and is the inspiration for the common saying that topologists can't tell a coffee mug from a donut.

Answer: **torus** [prompt on donut before read, accept **ring torus** before read, accept **1-torus**]

7. A number is only an antimorph if the Pell equation has this property. A Lie group has this property if it is connected and its Lie algebra commutator series eventually vanishes. The Feit-Thompson theorem showed that every finite group of odd order has this property, and the fact that the symmetric group does not have this property for $n = 5$ and above is the reason that the ● Abel-Ruffini theorem holds. Galois's theorem states that an algebraic equation has this property if and only if its group also has this property. The halting problem has this property for machines with fewer than four states but does not have it in general. For ten points, identify this property held by equations to which one can find the roots.

Answer: **solvability** [accept word forms]

8. The fraction of the numbers less than x with this property which are prime is on the order of $\frac{\ln \ln \ln x}{\ln \ln x}$. Belphegor numbers are examples of this type of number which contain the string 666. Lychrel numbers are numbers which are not known to produce one of these numbers after repeated application of the reverse-then-add sequence; the smallest such number is 196. The first 9 Demlo numbers have this property because the nth Demlo number is the square of the nth repunit. All of these numbers with an even number of digits are divisible by ● 11. For ten points, identify this property held by numbers which are the same backwards as they are forwards.

Answer: **palindrome**s [accept word forms]

9. The Dirichlet distribution is the conjugate prior for the distribution named for this kind of data, and the probit regression technique is used for a subset of this kind of data. The techniques of dummy coding, effects coding, and contrast coding are used to analyze this kind of data, which is described by distributions also termed generalized ● Bernoulli distributions. This kind of data is termed dichotomous if it has only two possible values. The act of treating continuous data as if it were this kind of data is called discretization. For ten points, identify this type of statistical data exemplified by blood type or country of origin which can only take on one of a fixed set of values.

Answer: **categorical** data [accept "variables" for "data"]

10. *Pencil and paper ready.* The area of a regular pentagon with this side length is $\frac{5}{4\sqrt{5-\sqrt{5}}}$. This is the only real solution to the equation $0 = x^3 - x^2 + x - 1$. The integral from 0 to this quantity of $\sqrt{x}$ is equal to $\frac{2}{3}$. Tthis quantity times π is equal to the surface area of the sphere $x^2 + y^2 + z^2 = \frac{1}{4}$, and it is equal to the eccentricity of a ● parabola. A triangle with vertices at the origin, $(0, 2)$ and $(1, 2)$ has area equal to this number, and this number is the determinant of the matrix $\begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$. For ten points, identify this number, the only nonzero solution to the equation $x^2 = x$.

Answer: 1

11. The Calkin-Wilf and Stern-Brocot trees will eventually generate every element of this set. A triangle whose side lengths and area are both members of this set is called Heronian. Niven's theorem restricts the values of the sine of x if it, and $\frac{x}{\pi}$, are both members of this set, extensions of which are called ● algebraic number fields. Every field of characteristic zero has this set as a subfield. This set is countable, however, it is dense in the reals under the standard topology. For ten points, identify this set of numbers expressible as $\frac{a}{b}$ where a and b are integers.

Answer: **rational** numbers

12. Carathéodory names one of these constructs which is induced by a set function and restricted to a sigma-algebra; that one of these is not necessarily an extension of that set function. One of these is called sigma-finite if the space it acts on can be decomposed into countably many sets with a finite value of this. These constructs must satisfy non-negativity, the null empty set property, and countable additivity. A set is called ● null if this is equal to zero for it. Lebesgue names the standard type for the real numbers, of, for ten points, what kind of mathematical construct which loosely denotes the size of an object?

Answer: **measure** [prompt on size before read, do not accept "metric"]

13. A transformation is called ergodic if its invariant subsets all have this property. This term describes a fiber bundle isomorphic to the cross product of the base space and a fiber. A Brunnian link consists of a set of linked loops such

that every proper sublink has this property; the simplest example of one is the Borromean rings. The loop described by this adjective takes every point to its basepoint. Zeros of the Riemann zeta function lying at ● odd negative integers are described by this adjective. Topological spaces described by this adjective are also called indiscrete and contain only two open sets. For ten points, identify this adjective which describes the group with one element, or a proof which is easy.

Answer: **trivial** [accept word forms]

14. Douglas's lemma guarantees the existence of one of the polar type of these, which can act on any bounded linear operator between Hilbert spaces and, in that application, is a factorization into a partial isometry and a nonnegative operator. The singular value kind of this process outputs $U\Sigma V*$. Schur names one of these which uses a unitary ● matrix and its inverse as well as a Schur form. When A is positive definite, the Cholesky one of these of A is unique. The spectral theorem allows the existence of an eigen type of this. For ten points, identify this type of matrix process which has QR and LU types and consists of dividing a matrix into factors with special properties.

Answer: **decomposition**s

15. Haga's theorem shows how to use this process to generate an arbitrary rational number. Maekawa's theorem relates the number of "mountains" and "valleys" obtained through this process. The Huzita-Hatori axioms formalize what can be done using this process, which can solve third-degree equations and therefore is capable of doubling the cube and trisecting the angle. ● Kawasaki's theorem governs this process by showing that alternating angles around a vertex must sum to 180 degrees, and it also determines when the outcome of this process can be flattened. For ten points, identify this Japanese practice which can be used for geometric construction as well as for art to create things like paper cranes.

Answer: **origami** [accept descriptive answers like **paper folding**, prompt on folding]

16. *Description acceptable.* A 2012 paper by Steffen and Hotchkiss experimentally compared several different methods of doing this, including the Wilma and Reverse Pyramid methods. A paper by Daniel Helkey uses a modeling approach to determine how luggage affects this process, and a 2013 article in the Economist discusses the benefits of the Flying Carpet method for doing this. Organizations like ● Frontier utilize the back-to-front method of doing this, while Southwest notably does not pre-assign seat locations. For ten points, identify this process which precedes an airline flight and which usually takes place on jetbridges.

Answer: **boarding** commercial **airplane**s [prompt on answers indicating just boarding with "what are they boarding?"]

17. Gårding's inequality gives a lower bound for one of these entities acting on $L\mathbf{u}$ and $\mathbf{u}$. Beauzam and Dégot's identity relates one of these which acts on $P \cdot Q$

and $R \cdot S$ to a sum; that one is the Bombieri one. If one of these constructs also defines a complete metric, the space it acts on is known as a ● Hilbert space. One example of these acts on two functions f and g and returns the integral of $f(x)g(x)$, and this operation performed on a and a is defined to be the norm of a. For ten points, identify this class of operations which generalize the dot product.

Answer: **inner product**s [generously prompt on dot products before read]

18. The algebra generated by these objects is isomorphic in two dimensions to the complex numbers and in three dimensions to the quaternions; in general it is the even subalgebra of the geometric algebra. These objects are isomorphic to skew-symmetric matrices, and are all simple in fewer than four dimensions. These objects' product is defined as the sum of their scalar interior product, their commutator product, and their order-4 exterior product. These objects have magnitute, attitude, and orientation and are generated by the exterior product of two ● vectors. For ten points, identify these extensions of vectors that can be interpreted as oriented plane segments.

Answer: **bivector**s

19. This question was answered affirmatively by Hilbert in 1909, but his proof was nonconstructive. Liouville showed that the fourth power case for this question had an upper bound of 53; Hardy and Littlewood showed that it was 19 for all sufficiently large integers, and that solution was extended to all integers in 1986. A special case of the solution to this problem can be proved using Minkowski's theorem; that special case is the Lagrange ● four-squares theorem. For ten points, identify this mathematical problem which asked of every natural number k whether there was a number s such that every integer can be written as at most s kth powers.

Answer: **Waring's problem** [accept **Hilbert-Waring theorem**]

20. This function has the property that the integral of the square of this function of x is equal to x times the square of this function of x minus twice x times this function of x plus $2x$. The integral of this function over x is equal to this function of the absolute value of this function of x, and ● $\frac{d}{dx}$**THIS** $(\cos x) = -\tan x$. A Taylor series for this function is **THIS**$(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$; this is also known as the Mercator series. This function is defined as $\int_1^x \frac{1}{x}$. For ten points, identify this inverse of the exponential function.

Answer: **natural log**arithm [accept answers including a variable, accept **ln**, prompt on logarithm