

Math Monstrosity, Packet 7

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.

If a question begins with “paper and pencil ready”, it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.

If, at any time during an equation, you see something like $\frac{\mathbf{THIS}}{2}$ or $\mathbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as “this function” or “this quantity” or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say “this thing” or “this”.

Pronunciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.

Please read Greek letters as they are (for example, read ϕ as “phi” and not “the golden ratio”, even if it represents the golden ratio), with the notable exception of \sum and \prod , as in $\sum_{n=1}^5$, which should be read as “the sum from $n = 1$ to 5 of”.

Similarly, \int_a^b is “the integral from a to b ” and $\lim_{n \rightarrow \infty}$ is “the limit as n approaches infinity”.

In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as “ f of x ” and “lambda of u and v ” respectively, and not as “ $f x$ ” and “lambda $u v$ ”.

Please read large and/or complex fractions by saying “in the numerator”, reading the numerator, saying “in the denominator”, reading the denominator, and then saying “end of fraction”. For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read “ a over b squared plus c ”.

Please read $\binom{a}{b}$ as “ a choose b ”, not as “ a over b ”.

If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means “the transpose of A ”, read it as “ A to the power of T ” or “ A superscript T ”. Most of the notation used in this tournament is common enough that such descriptions, using words like “subscript” and “superscript”, should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. Real normed algebras are also described using this term, which also describes a series of nested normal subgroups of a finite group. This term is used to describe a partition in which order matters, and transformation monoids are defined by this operation. Faà di Bruno's formula is used to compute the n th derivative of an expression defined by this operation. The iterated version of this operation is referred to as the functional \bullet power, and this is the group operation used to combine permutations in the symmetric group. For ten points, identify this binary operation which acts on functions and is denoted by a small circle.

Answer: function **composition**

2. The octahedral group has this order, and the largest Mathieu group is designated with this subscript. The 4-dimensional regular polytope with this many cells is self-dual, is constructed of octahedral cells, and has no direct 3-dimensional analogue. An arrangement of unit spheres in this many dimensions can be constructed using the binary Golay code, uniquely solves the kissing number problem, and is known as the \bullet Leech lattice. This number, which is divisible by every positive integer less than its square root, is one less than a perfect square and three less than a perfect cube. For ten points, identify this number, equal to four times the smallest perfect number as well as itself being equal to $4!$.

Answer: 24

3. This system of geometry's distance metric can be combined with the Euclidean metric to define the akritean distance. The two-dimensional version of this system of geometry is equivalent through rotation and scaling to that defined through Chebyshev distance. In this system of geometry, ellipses whose foci have different x - and y - coordinates are shaped like Euclidean octagons. Spheres in this system of geometry are shaped like Euclidean octahedra, while circles are shaped like rotated Euclidean \bullet squares. For ten points, identify this system of geometry in which the distance between two points is the sum of the absolute differences of their coordinates.

Answer: **taxicab** geometry [accept **Manhattan** geometry, **snake** geometry, ℓ_1 geometry, **rectilinear** geometry, or **city block** geometry]

4. Tarski names a type of group also described by the same term as this group which is an infinite group G such that any proper subgroup of the group is a cyclic group with order a fixed prime number. Prime numbers which divide the order of this group are called supersingular; there are 15 such primes. A certain collection of groups is divided into "pariahs" which are not subgroups of this one, and the "happy family" which is. This group can be built from two out of three of the Fischer group Fi_{24} , the Conway group Co_1 , and its own \bullet "Baby" version. This group was initially constructed as the automorphism group of

a 196,884-dimensional algebra. For ten points, identify this largest of the 26 sporadic finite simple groups.

Answer: **Monster** group

5. This novel's protagonist describes Conway's soldiers as a good problem to do in your head because "you can make it as complicated as you need to fill your brain by making the board as big as you want and the moves as complicated as you want". A story involving a mathematician, an economist, and a logician describing a cow, and the mathematician's precise use of the phrase "at least one", is one of the three jokes known to the protagonist of this novel, who recites the cubic numbers to calm himself down and prevent himself from harming others. The chapters of this novel are numbered by successive \bullet prime numbers. Christopher John Francis Boone is the protagonist and narrator of, for ten points, what novel by Mark Haddon?

Answer: The **Curious Incident of the Dog in the Night-Time**

6. A version of Runge's theorem for these objects can be found by setting $P = \infty$, and the Bombieri inner product acts on these objects. The Christoffel-Darboux formula gives an identity relating to a series of these objects. The Lukács theorem provides a way to represent nonnegative instances of these objects, and the Nullstellensatz concerns the ideals of rings of these objects. Weierstrass's theorem on these objects states that they can be used to \bullet approximate any continuous function over a finite closed interval. The fundamental theorem of algebra is a statement about the roots of, for ten points, what algebraic objects which consist of sums of powers of variables?

Answer: **polynomials**

7. Dixon names a test for identifying these things which he says should not be used more than once. These things, which come in point, collective, and contextual varieties, can be detected by Grubb's test. Chauvenet names a criterion for determining whether these things should be taken into account, and an earlier criterion for that is named for \bullet Peirce. Two methods for dealing with these things are trimming and Winsorising. The median is preferred to the mean for determining central tendencies because it is less affected by these. For ten points, identify these data points which are far away from the rest of the data in a data set.

Answer: **outliers**

8. A complex continuous version of this question involves a fractal graph which encodes discrete information about this question's traditional statement. A 1981 paper by Lynn Garner showed that a negative answer to this question must involve thousands of terms. Paul Erdős declared about this question that "mathematics may not be ready for such problems". The generalized version of this conjecture includes negative numbers and involves four terminal \bullet cycles rather than just one. This question asks whether the iterated Syracuse function of every integer is equal to one for sufficiently many iterations. For ten

points, identify this conjecture in number theory which asks whether iterating the operation of multiplying odd numbers by 3 and adding 1 and dividing even numbers by two will always lead to the cycle (1, 2, 4).

Answer: Collatz conjecture [accept **3n+1** conjecture until “3” and prompt after, accept Ulam conjecture, Kakutani’s problem, Thwaites conjecture, Hasse’s algorithm, the hailstone problem; accept Syracuse problem before “Syracuse” is read and prompt after]

9. Russell Rogers and Leonard D’Andrea were granted a patent for these things under the name “Changeable Amusement Devices and the Like.” Les Pook has written two books about these things, including *Serious Fun with [these things]* and *[These Things] Inside Out*. The Tuckerman traverse is a technique which is used to expose all of one of these things. The “hexahexa” type of these things has six hexagonal faces, and is ● folded from a strip of paper divided into eighteen equilateral triangles. The subject of Martin Gardner’s first-ever *Mathematical Games* column was, for ten points, what objects which can be folded to reveal faces which were not originally shown on their back or front?

Answer: flexagons [accept specific flexagons like “hexahexaflexagon”]

10. Hurwitz names a theorem which bounds the accuracy of approximations of numbers with this property, and Segre’s Theorem generalizes that theorem. The signature sequences of these numbers are fractal sequences. When viewed from the origin with normal perspective along the line $y = x$, a quadrant of Euclid’s Orchard will resemble the graph of a function continuous only at these numbers; that is ● Thomae’s function. If the roots of a monic polynomial are not integers, then they must have this property. For ten points, identify this property held by square roots of numbers that aren’t perfect squares, and more generally by all numbers which can’t be written as fractions.

Answer: irrational numbers

11. The left or sub- version of this function counts the number of derangements of n objects. A conjecture of Erdős states that there are only three pairs of numbers (m, n) such that $\mathbf{THIS}(n) + 1 = m^2$. The Pochhammer symbol is also known as this function’s ● “rising” type. The exponential type of this function is defined by the recurrence relation $a_n = n^{a_{n-1}}$. This function’s double version is actually smaller because it only includes odd or even numbers in the computation of its output. This function of n gives the number of permutations of n objects. For ten points, identify this function denoted by an exclamation point.

Answer: factorial

12. *Pencil and paper ready*. This number represents the expected value of the determinant of a 5×5 matrix in which every element is randomly chosen from the set $\{-1, 0, 1\}$. If Alice and Bob each draw five cards from a deck of ten cards numbered one through ten, this number represents the number of distinct ways that their decks can sum to the same number. ● If dice are constructed

in the shape of each Platonic solid, and their faces are numbered in ascending order starting with one, this is the number of ways to roll a five on every die. For ten points, identify this number, the probability of rolling a sum of 13 with two standard dice.

Answer: 0

13. A theorem named for this mathematician states that polyhedra with congruent corresponding faces are congruent. An equation named for this mathematician has a set of solutions which was found by Georg Hamel, and describes ● additive functions. This mathematician names a quantity that is the convolution of two power series -- his "product". The theorem stating that the contour integral of a complex-valued holomorphic function is zero is called this mathematician's integral theorem. This mathematician, with Schwartz, names an inequality relating to inner products. For ten points, identify this French mathematician, whose namesake sequences all converge in the real numbers.

Answer: Baron Augustin-Louis Cauchy

14. The Schur multipliers of the fourth and fifth of these groups, as well as the eighth and beyond, are the cyclic groups of order two, while the Schur multipliers of the sixth and seventh are the cyclic groups of order six. These groups are associated with graphs named for them, and the fourth of those graphs is the cuboctahedral graph. The icosahedral group is equal to the direct product of the fifth of these groups and the second cyclic group. If n is greater than 4, then the n th one of these groups is simple. The group order of the n th of these groups is ● $\frac{n!}{2}$. For ten points, identify these groups of all even permutations of a certain size.

Answer: **alternating** groups

15. A mathematician who co-names a theorem about convergent sequences with Weierstrass also held this non-academic occupation. The developer of the mean speed theorem and a member of the Oxford Calculators, Thomas Bradwardine, held this occupation and was elevated to a very high position within it shortly before his death. The mathematician who discovered the determinant form for the higher-order chain rule, and after whom it is named, held this occupation and was ● beatified as a result of his work in this occupation. Marin Mersenne notably held, for ten points, what general occupation of formal involvement with religion?

Answer: Roman Catholic clergy [accept monks or priests or other specific clerical occupations]

16. In the equilateral kind of these objects, extending two line segments will result in a 60-degree angle. The tangential kind of these objects is distinguished by possessing an incircle, and the area of a tangential one of these objects is equal to the inradius times the semiperimeter. Bretschneider's formula is an analog of Heron's formula for computing the area of these objects, and ● Brahmagupta's formula is a special case used to compute the area of the cyclic kind of these

objects, which are governed by Ptolemy's theorem. All of these objects tile the plane, and lozenges and kites are special cases of, for ten points, what class of geometric shapes which include trapezoids and rectangles?

Answer: **quadrilaterals** [antiprompt on specific types thereof]

17. A local surface is a mapping from an open subset of the Cartesian plane to \mathbb{R}^n that has this property, and all topological manifolds on three dimensions or fewer have this property. Functions that have this property only on a set of Lebesgue measure zero include one named for Blancmange. A function is complex **analytic** if it has this property at every point in the complex plane, and any function with this property is also continuous. For ten points, identify this property held by a function f at a point x if the limit as h goes to zero of $\frac{f(x+h)-f(x)}{h}$ exists at that point.

Answer: **differentiability** [accept word forms]

18. The original definition of topological spaces included as an axiom that all spaces have this property, which a space has if it is both preregular and Kolmogorov. A space is said to be completely or functionally this if any two points in it can be separated by a continuous function. If a space has this property, then limits of nets and filters on it are unique, and the intersection of all closed neighborhoods of x is equal to the **singleton** $\{x\}$. This property's namesake also notably names a fractal dimension. For ten points, identify this property held by a topological space if any two distinct points in it have disjoint neighborhoods.

Answer: **Hausdorff** property [accept things like **Hausdorff space**, accept **separatedness** or T_2 until "namesake" and prompt afterward]

19. An extension of this theorem can be used to generalize the Stolarsky mean; that extension is called this theorem for divided differences. This theorem can only be applied to the real and imaginary parts separately for complex functions. **Cauchy** names an extension of this theorem which applies to parametric curves, and Rolle's theorem is a special case of this theorem. For ten points, identify this result which states that for a continuous function f on a closed interval $[a, b]$, there exists a point c between a and b such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Answer: **mean value theorem**

20. One condition for a submersion to be smooth is that the differential have this property. One property required of gerbes is that there must be a local homeomorphism with this property whose accompanying category is non-empty. If a certain embedding from a normed space to its second dual vector space has this property, then that space is reflexive; the embedding in question is the canonical map. There exists a map with this property from any **covering space** to the space it covers. A map with this property is called an epimorphism. The term onto is used to describe, for ten points, what sort of functions which map to the entire range?

Answer: **surjective** [accept **onto** until read]