

Math Monstrosity, Packet 6

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.

If a question begins with “paper and pencil ready”, it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.

If, at any time during an equation, you see something like $\frac{\mathbf{THIS}}{2}$ or $\mathbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as “this function” or “this quantity” or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say “this thing” or “this”.

Pronunciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.

Please read Greek letters as they are (for example, read ϕ as “phi” and not “the golden ratio”, even if it represents the golden ratio), with the notable exception of \sum and \prod , as in $\sum_{n=1}^5$, which should be read as “the sum from $n = 1$ to 5 of”.

Similarly, \int_a^b is “the integral from a to b ” and $\lim_{n \rightarrow \infty}$ is “the limit as n approaches infinity”.

In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as “ f of x ” and “lambda of u and v ” respectively, and not as “ $f x$ ” and “lambda $u v$ ”.

Please read large and/or complex fractions by saying “in the numerator”, reading the numerator, saying “in the denominator”, reading the denominator, and then saying “end of fraction”. For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read “ a over b squared plus c ”.

Please read $\binom{a}{b}$ as “ a choose b ”, not as “ a over b ”.

If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means “the transpose of A ”, read it as “ A to the power of T ” or “ A superscript T ”. Most of the notation used in this tournament is common enough that such descriptions, using words like “subscript” and “superscript”, should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. If a coordinate plane is defined with both axes ranging over the limited version of this set, then the parallel postulate is violated. One function defined on this set is the standard part function, which returns a real number which is an element of the monad of the input; that function is a ring homomorphism from the limited version of this set to the real numbers. This set can be constructed through ultrapowers on real sequences, and this set is subject to the transfer principle, which means statements in first-order logic true about the real numbers are true about it. ● Non-standard analysis is the study of, for ten points, what extension of the real numbers which includes infinitesimals?

Answer: hyperreal numbers [accept non-standard reals until “non-standard”]

2. Friedholm names a generalization of this to operators, while Dieudonné names a generalization to non-commutative spaces. Another generalization of this is called the Berezinian and is used on supermanifolds. Jacobi’s formula is used to differentiate this, and Sylvester’s theorem relates two instances of this to each other. If this quantity for some object is equal to 1 or -1 , then that object is referred to as unimodular. This ● alternating form is also a multiplicative map, and the Cayley-Hamilton theorem relates this to a polynomial. Laplace’s formula for calculating this quantity involves multiplying the input by its adjugate. For ten points, identify this quantity that is zero for non-invertible matrices.

Answer: determinant

3. The manipulation of these objects in order to produce certain expressions is known as index gymnastics. Riemann and Christoffel name one of these objects used to express the curvature of manifolds. Contraction is an operation reducing the total order of these objects by two, and the most commonly used notation for these objects is Ricci calculus. Bilinear forms are examples of these objects which also include bivectors and linear transformations. One of these objects named for Cauchy is used to describe the ● stress of a material, and these objects describe linear relations like the inner product and cross product. For ten points, identify these objects which generalize vectors and matrices.

Answer: tensors

4. A 1968 paper by Chorin presents a finite-difference method for numerically approximating a solution to these things. A book titled after these things by Roger Temam includes an appendix about the curl operator and chapters about their steady-state and evolution type. These things are distinguished from the Euler equations because they account for situations which are not ● inviscid. One of the Millenium problems asks whether a smooth solution exists to the three-dimensional version of these equations. For ten points, identify these doubly-eponymous differential equations which describe the motion of fluids with nonzero viscosity.

Answer: Navier-Stokes equations [you don’t need “equations” after it’s read]

5. A vector on a Hilbert space is described by this term if there exists a bounded linear operator such that the orbits of the operator on the vector are dense in the Hilbert space. A group is called virtually this if it contains one of these groups, and that group has a finite number of cosets. For every n , there exists exactly one of these groups with n elements, and the only \bullet simple Abelian groups are the instances of these groups with non-composite order. These groups correspond to the rotational symmetries of a polygon. For ten points, identify this class of groups with one generator of which every element is a power.

Answer: cyclic groups

6. The second edition of a work of this name added an appendix describing the Sheffer stroke. A translation of one work of this name was performed by Andrew Motte and was modernised by Florian Cajori. The first proposition in one work with this name defines the sign which represents implicatures. The third section of one work of this name includes a description of the variation of the Moon's orbit. The Information Processing Language was first utilized to computationally demonstrate the theorems in one work of this name. \bullet "De motu corporum" titles the first section of a work with, for ten points, what name shared by Russell and Whitehead's axiomatic construction of set theory?

Answer: Principia Mathematica

7. Simon Stevin proved this result for polynomials by constructing the decimal expansion of a solution, and a function is said to be Darboux if it fulfills the property laid out in this result. Bolzano's theorem is equivalent to this result, which is true because continuous functions take \bullet connected sets to connected sets. This theorem fails to be true for the rationals because they are not connected; for example, although there exist rationals a, b such that $a^2 = 0$ and $b^2 = 4$, there exists no rational c such that $c^2 = 2$. For ten points, identify this result from real analysis stating that any function from the interval $[a, b]$ to the real line must assume every value between $f(a)$ and $f(b)$.

Answer: intermediate value theorem [accept Bolzano's theorem until read]

8. This man designed a large ship, named after his home, which only sailed once. In Cicero's *De re publica*, Marcus Claudius Marcellus takes to Rome two devices built by this man which show the movements of the Sun, Moon, and planets. This man wrote a treatise on the behavior of right paraboloids in various fluids, *On \bullet Floating Bodies*. A palimpsest named for this man contains his *The Method of Mechanical Theorems*, in which he describes his methods for solving problems with a procedure based on centers of mass. *The Quadrature of the Parabola* was written by, for ten points, what ancient mathematician who lived in Syracuse and invented a namesake screw?

Answer: Archimedes

9. Dini's theorem says that if something has a weak form of this property and fulfills other conditions, then it also has a stronger form of this property. Bertrand

and Dirichlet both name conditions to determine whether something has this property. Hadamard's theorem gives a means for determining this quantity's namesake ● radius. If a series has the conditional variety of this property, then its sum can be changed by rearranging its values. The harmonic series notably does not have this property, although if one removes all the numbers which have the digit nine somewhere in the denominator, the resulting series does. For ten points, identify this property held by a series or sequence whose limit at infinity exists.

Answer: **convergence** [accept word forms]

10. One common source of inconsistencies in this concept is the axiom schema of unrestricted comprehension, which leads to things like the Burali-Forti paradox and Curry's paradox. Paul Halmos wrote a book titled after this concept, although the book does not describe this concept because it presents an axiomatic construction of its subject. ● Russell's paradox naturally arises in this system, and more advanced systems that replaced this one include one named for Zermelo and Fraenkel. For ten points, identify this system of set theory driven by an intuitional understanding and possessing no formal underpinning.

Answer: **naive set theory** [prompt on set theory before read]

11. *Description acceptable.* This object is the basis of a branch of projective geometry designed to give apartments and buildings equal standing. Durov constructed this object as a commutative algebraic monad, and Borger used descent to construct it from finite fields. Pointed sets are vector spaces over this object, and Weyl groups are simple algebraic groups over it. The integers are conjectured to form an algebra over this object, which could allow a proof of the ● Riemann hypothesis. Any finite set is both an affine space and a projective space over this object. For ten points, identify this hypothetical algebraic object which has characteristic one.

Answer: **field with one element** [accept anything indicating a **field** with **characteristic one** before read, accept F_1 , accept F_{un} , accept anything like "the field where zero equals one"]

12. Barth names a surface of this degree having the maximum number of ordinary double points, and Cayley names a curve of this degree in Cartesian coordinates which is given in polar coordinates by $r = 4a \cos^3\left(\frac{\theta}{3}\right)$. The Franklin graph has a chromatic number equal to this value. An analog to the four exponentials theorem with this many exponentials was proven by Siegel, Schneider, Lang, and Ramachandra. The ● Feynman point is a term used to refer to the first sequence of this many 9's which occurs in the decimal expansion of π . This quantity is equal to the order of the smallest non-Abelian group. For ten points, identify this smallest perfect number.

Answer: 6

13. An array with this property is related to infinitive sequences and ranges through all of the natural numbers. The Wolfram Language attribute for this

property is “Flat”. The Bol loop has the power type of this property, though it does not have this property in general. A semigroup is a groupoid that has been additionally endowed with this property. The ● octonions are alternative but do not have this property, while the quaternions have this property but not commutativity. For ten points, identify this property which allows parentheses to be rearranged.

Answer: **associativity** [accept word forms]

14. The Shah distribution consists of the sum of equally spaced horizontal transformations of this function, which is also known as its namesake’s “comb”. The Sokhotski-Plemelj theorem relates this function to the Cauchy principle value of the reciprocal function. This function can be expressed as the derivative of the Heaviside step function. This function is the limit of a sequence of strongly peaked functions, and ● Kronecker names a discrete analog of this function. The integral of this function over an interval is zero unless that interval contains the origin, in which case it is one. For ten points, identify this function equal to zero everywhere but the origin, at which it is equal to infinity.

Answer: **Dirac delta function** [prompt on delta function]

15. *Description acceptable.* The proof of this result begins by stating “First, let π be some k -cycle on $[n]$ ”. Ken Keeler, who originated this result, stated in a DVD commentary that he didn’t feel it was of sufficient importance to be considered a “theorem”. This result’s proof is reliant on the fact that any permutation can be expressed as a sum of disjoint cycles, because the device to which it pertains cannot perform the same switch twice. This result was proved by Ethan Tate and Clyde Dixon of the ● Harlem Globetrotters, who then successfully applied it to save characters like the Professor and Leela. Appearing in the episode “The Prisoner of Benda”, for ten points, identify this result which guarantees that people can always be restored to their bodies with the aid of two outside parties.

Answer: the **body-switching** theorem from **Futurama** [prompt on answers only suggesting body-switching after “bodies” is read and accept any answer suggesting either underlined part beforehand; accept any answer mentioning Ken **Keeler** before read]

16. This operation, when applied to a one-form, gives a result described by the comma derivative. The conjugate this method, an algorithm for finding the local minima of a function, is only possible if this operation can be computed. The convective operator is defined as a product of a vector field and the symbol for this operation. The symbol used to describe this operation, when squared, instead describes the ● Laplacian; that symbol is known as “nabla”. A vector field is known as “conservative” if it can be expressed as this operation of another field. For ten points, identify this operation from multivariable calculus, which expresses the “slope” of a multidimensional scalar field and is symbolized with an upside-down delta.

Answer: **gradient** [accept nabla before read, accept del]

17. The number of states of an $n + 1$ -flexagon is equal to the n th member of this sequence. The super variant of this sequence counts lattice paths with diagonal steps, as opposed to this sequence, which counts p -good paths. This sequence obeys the recurrence relation that the n th element of this sequence is equal to the sum over the first $n - 1$ elements of $S_k S_{n-k-1}$, and the n th element of this sequence gives the number of binary bracketings of $n + 1$ letters. The n th element of this sequence gives the number of ways that a regular n -gon can be divided into $n - 2$ \bullet triangles if orientation matters. For ten points, identify this sequence named for a Belgian mathematician whose n th element is equal to $\frac{1}{n+1}$ times $2n$ choose n .

Answer: Catalan sequence

18. Schroepel counted how many order-5 examples exist of these things, and Frénicle de Bessy enumerated the 880 of these things of order 4. These things were assigned to each of the seven planets in works like the Liber de Angelis. The Kuberkolam is a term used in India to refer to one of these things, which can be generated by the Siamese method. One of these things appears below a bell and to the right of an hourglass in a \bullet Dürer engraving entitled Melencolia I. The Lo Shu is an example of, for ten points, what square arrays of numbers whose rows, columns, and main diagonals all sum to the same number?

Answer: magic squares

19. A blog post by Philippe Guglielmetti used this thing to determine which numbers are “interesting” and which are not. This entity uses the keyword “**dumb**” to describe things which are “unimportant” or do not relate to mathematics, and it uses “**cofr**” to describe continued fraction expansions. One entry in this entity consists of the n th \bullet term of the n th entry in this entity. The 100,000th entry in this entity counts the number of marks on the Ishango bone, while the first counts the number of groups of order n . For ten points, identify this database created by Neil Sloane which catalogues certain lists of numbers.

Answer: On-line Encyclopedia of Integer Sequences

20. The Risch algorithm is an algorithm used to determine whether this action will give a result consisting of elementary functions. One approximate method for performing this action is Romberg’s method, which repeatedly applies Richardson extrapolation and is a Newton-Cotes formula. An archaic term for the approximate form of this action is \bullet “quadrature”, and one well-known approximate method for doing it is the Monte Carlo method. The “contour” form of this process is performed on a complex-valued function over a curve in the complex plane. The disk and shell methods are used for, for ten points, what operation which can be performed by parts or by u -substitution?

Answer: integration [accept word forms]