

Math Monstrosity, Packet 5

Conor Thompson
University of Michigan

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.

If a question begins with “paper and pencil ready”, it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.

If, at any time during an equation, you see something like $\frac{\mathbf{THIS}}{2}$ or $\mathbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as “this function” or “this quantity” or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say “this thing” or “this”.

Pronunciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.

Please read Greek letters as they are (for example, read ϕ as “phi” and not “the golden ratio”, even if it represents the golden ratio), with the notable exception of \sum and \prod , as in $\sum_{n=1}^5$, which should be read as “the sum from $n = 1$ to 5 of”.

Similarly, \int_a^b is “the integral from a to b ” and $\lim_{n \rightarrow \infty}$ is “the limit as n approaches infinity”.

In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as “ f of x ” and “lambda of u and v ” respectively, and not as “ $f x$ ” and “lambda $u v$ ”.

Please read large and/or complex fractions by saying “in the numerator”, reading the numerator, saying “in the denominator”, reading the denominator, and then saying “end of fraction”. For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read “ a over b squared plus c ”.

Please read $\binom{a}{b}$ as “ a choose b ”, not as “ a over b ”.

If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means “the transpose of A ”, read it as “ A to the power of T ” or “ A superscript T ”. Most of the notation used in this tournament is common enough that such descriptions, using words like “subscript” and “superscript”, should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. One commentary on this work stated that it “[brought] to irrefragable demonstration the things which were only somewhat loosely proved by [its author’s] predecessors”. A fragment of this work was recorded on Oxyrhynchus papyrus 29, and this work was translated from Arabic to Latin by Adelard of Bath. Carl Boyer estimated that the only work with more editions than this one was the Bible. Abraham ● Lincoln kept a copy of this work in his saddlebag, and Edna St. Vincent Millay wrote that this work’s author “alone has looked on Beauty bare”. For ten points, identify this seminal geometry treatise, consisting of thirteen books and written by Euclid.

Answer: Euclid’s Elements [accept Stoicheia]

2. Golomb constructed a combinatorial proof of this result which used the counting of necklaces. The converse of this result is false, but it is strengthened by Lehmer’s theorem, which is true. This result generalizes the Chinese hypothesis and is itself a generalization of Euler’s totient theorem. ● Carmichael numbers are counterexamples to the converse of this result for all bases, and numbers which are counterexamples to its converse for some base are called pseudoprimes. For ten points, identify this result stating that for a natural number a and a prime number p , then a^p is congruent to $a \pmod{p}$; it is contrasted with its namesake’s “last” theorem.

Answer: Fermat’s little theorem

3. Gelfond’s theorem gives conditions for the number a^b to have this property, and the Gelfond-Schneider constant and Champernowne’s constant have this property. A function has this property if it cannot be written as a finite sequence of addition, multiplication, and root extraction. ● Liouville’s constant, whose decimal expansion has a zero in the n th decimal place unless n is a factorial number, was the first number proven to have this property. Hermite’s theorem established that this property is possessed by e . For ten points, identify this property held by numbers which are not the solution to any polynomial equations.

Answer: transcendental [accept word forms]

4. A vector field on a manifold M is called nice if it has one of these C such that the intersection of C with any leaf in the interior of M is a singleton. The elements of these objects correspond to the cosets of isotropy subgroups. For the orthogonal group of signature $(1, 1)$, these are the origin, the four axes, and the hyperbolas defined by $xy = k$. An action is called a ● stabilizer for x if this object for x has only one element under that action. For ten points, identify this concept from group theory which for an element x of a set consists of the set of gx for all g in the group acting on that set, described by a term which also describes the path taken by a satellite.

Answer: orbit

5. Kiyosi Itô included an axiom named for this entity in his treatment of ZFC set theory. A set function μ^* is said to be an outer measure if it is countably monotone and μ^* of this object takes on a certain value, and for any presheaf F on a topological space, F of this object must take on that same value. The interior of certain subsets of Baire spaces is equivalent to this object. This object may result from infinite intersection of \bullet unbounded nested closed sets, but not from infinite intersection of bounded nested closed sets. This set is always clopen in every topology. For ten points, identify this set whose complement is the universal set and which has no elements.

Answer: empty set [prompt on null set since that's just a set with measure zero, accept "set with no elements" until read]

6. If ω is taken to be the set of natural numbers, then this entity $\text{sub } \omega$ is a model for set theory without the axiom of infinity, while this entity $\text{sub } \omega + \omega$ is a model for Zermelo set theory. This entity is constructed by transfinite induction over the ordinal numbers. This entity is defined to be the union of all stages, which consist of all sets with the same rank and form the cumulative \bullet hierarchy. For ten points, identify this entity symbolized V , the class of hereditary and well-founded sets and foundation for modern set theories.

Answer: von Neumann universe [accept von Neumann hierarchy of sets before "hierarchy" and prompt afterward, prompt throughout on just hierarchy of sets]

7. Every metric space with this property is a Baire space by the Baire category theorem, and the Banach fixed point theorem ensures that a contraction mapping on a metric space with this property admits a fixed point. A metric space has this property if and only if every possibly infinite set of nested closed subsets has a nonempty intersection. The space of p -adic numbers has this property for any prime p . A metric space is considered to have this property if every \bullet Cauchy sequence in that metric space converges. For ten points, identify this property held by the real numbers because there are no gaps in the number line.

Answer: completeness [accept word forms]

8. Bing's theorem states that if M fulfills certain conditions, and every one of these in M is contained in the interior of a ball in M , then M is isomorphic to a hypersphere. A frame is one of these which is associated with a knot. A sutured manifold is a manifold endowed with a set of these which divide it into positive and negative pieces, and the Schönflies theorem states that the closure of one of the disconnected components of \mathbb{R}^2 minus one of these is homeomorphic to the unit 2-ball. If these are \bullet simple, then they are named for Jordan. For ten points, identify this kind of curve which has no endpoints and encircles an area.

Answer: closed curve [accept Jordan curve until "Jordan" is read and prompt afterwards, prompt on curve]

9. Baxter's four-coloring constant is given for a lattice described by this adjective. The distribution function of a distribution described by this term consists

of two parabolic arcs, and the probability density function of that distribution consists of two lines. Matrices with the strict version of this property are **●** nilpotent, and matrices described by this adjective have determinants equal to the product of the numbers on their diagonal. Numbers described by this adjective are of the form $\binom{n+1}{2}$, and are generated by adding all integers up to a certain point. For ten points, identify this adjective, which can also describe things which resemble 3-sided polygons.

Answer: **triangular** [accept word forms]

10. Netto's conjecture states that the probability that any two elements of the n th of these groups generate the group goes to $3/4$ as n goes to infinity. Jordan's theorem on these groups states that a primitive subgroup of the n th of these groups is equal to either that group or the n th **●** alternating group if it contains a q -cycle. These groups are transitive, and the number of conjugacy classes of the n th of these groups is equal to the partition function of n . These groups are generally non-abelian, and the n th of these groups is a permutation group of order $n!$. For ten points, identify these groups, the n th of which is the group of permutations with n elements.

Answer: **symmetric** groups

11. The narrator describes one location in this work by declaring "Neighbors remained neighbors till death did them part". The presence of Fog, which causes far-away objects to appear dimmer, is central to the art of Sight Recognition in this work. One conflict in this work between Pantocyclus and Chromatistes leads to the passage of the Universal **●** Colour Bill. The protagonist of this work uses the phrase "Upward, not Northward" to attempt to convince his compatriots of the existence of a third dimension. For ten points, identify this satire of Victorian society by Edwin Abbott Abbott, which features the protagonist, A Square, who lives in the title two-dimensional world.

Answer: **Flatland**: A Romance of Many Dimensions

12. The term "cathetus" is an archaic term for certain parts of these objects. Fermat's theorem about these shapes says that their area cannot be a perfect square unless at least one side has irrational length; numbers that can be the area of a rational-sided one of these shapes are called congruent. Polyaboloes consist of special cases of these shapes joined at their edges, and one of these shapes is called primitive if its side lengths are relatively prime. The midpoint of the longest side of one of these shapes is equidistant from all of its **●** vertices, and if one of these shapes is inscribed in a circle then its longest side is a diameter. For ten points, identify these shapes whose side lengths obey the Pythagorean Theorem.

Answer: **right triangles** [prompt on triangles]

13. Wilson names one of these for a prime p which must be divisible by p for p to be a Wilson prime. Grün's lemma states that the center of one of these of

a perfect group is trivial. Any local ring only has one *this* ring which is a field; that is called the residue field. The cokernel of a group homomorphism is this of its image on the target group. A ring described by this term is obtained by ● partitioning the ring into equivalence classes based on an ideal of that ring. For a group G and normal subgroup H , the this group of H in G is the group of cosets of H . For ten points, identify this term used to refer to the result of a division operation.

Answer: **quotient**

14. A function determines one of these if it satisfies the properties of homogeneity, bisymmetry, and cyclicity. One of these is called major if the function which determines it takes only one argument. One of these called the mittenpunkt is the center of a Mandart inellipse, and another named for Lemoine is the intersection of the symmedians. The nine-point one of these is defined as the center of the nine-point circle, and the ● Euler line passes through several of these. One of these is the intersection of the altitudes and is the “ortho” one. For ten points, identify these special points with relation to certain shapes, which are exemplified by one prefixed “circum-” and the centroid?

Answer: **triangle centers** [prompt on centers, antiprompt on any specific triangle center]

15. A reddish-brown one of these objects appears in the background of the scene in the Simpsons episode “Treehouse of Horror VI” where Homer discovers the third dimension. Jim Arvo and Dave Kirk produced a ray-traced image showing six stone columns, on five of which are the Platonic solids and on the sixth of which is one of these objects; that image is titled “The Six Platonic Solids”. The mathematical coordinates describing one of these objects were made available by Martin ● Newell, and many graphics engines have one of these non-geometric objects as a graphics primitive. For ten points, identify this household item commonly used as a test or demonstration in computer graphics.

Answer: Melitta **teapot**

16. This process is performed with respect to the Jordan measure, and can only be performed on an indicator function of a set if that set is Jordan measurable. This process is generalized by a similarly named one called the “gauge” one, and it cannot be performed on any function which has more than countably many points at which it is discontinuous. This process is equivalent to a similar process named for ● Darboux, and the Lebesgue monotone convergence theorem fails to hold for this process. This process uses tagged partitions in order to compute intermediate values, which are called “sums” and named for the same person as this process. For ten points, identify this first rigorously-defined integral.

Answer: **Riemann integral** [accept word forms, prompt on Darboux integral before “Darboux”, prompt on integral before read]

17. The Tschirnhaus transformation and Cayley’s resolvent are tools used in performing this action, another variant of which can be performed using Lill’s

method. Tartaglia and Fiore famously had a series of contests in which both displayed their ability to perform this action, and later, Tartaglia had another competition with Ferrari. The Jenkins-Traub method is an approximate method for performing this action, and another approximate method uses ● Vieta's formulas. Évariste Galois showed how to decide whether it is possible to perform this action, which Abel and Ruffini both proved was impossible in the general case. For ten points, identify this action in which the solutions to certain equations are found.

Answer: **solving polynomials** [accept logical equivalents, accept "finding the roots of a polynomial", antiprompt on "solving quadratics" or "solving cubics" or similar]

18. A paper by Morris entitled "An Experimental Analysis of the Effectiveness of Features in [these things]" concluded that perception of them was serial and not pre-attentive. A classic paper in the Journal of Marketing by Huff describes these things' use for representing data. Edward Tufte stated that a graph containing these things would "reduce well, maintaining legibility", and argued that the use of these things represented the "limit of graphical... eccentricity". Flury and Riedwyl proposed that these things be made asymmetric, which would allow features like ● eyes and mouths to carry twice as much information. For ten points, identify these means of graphically representing multivariate data, named for an American mathematician and consisting of stylized faces.

Answer: **Chernoff faces** [prompt on faces]

19. After rotation and reflection, there are this many triabolos and pentiamonds. Sylvester's problem of this many points asks the probability that this many randomly chosen points in a planar region are all located on the boundary of their convex hull. The differential equation named for phi and this number is expressed as $u_{tt} - u_{xx} + u + u^3 = 0$. A closed embedded smooth curve in the plane has at least this many extrema of curvature. Every positive integer can be written as the sum of this many perfect ● squares. In special relativity, vectors with this many dimensions are acted upon by Lorenz transformations. For ten points, identify this number, the largest number of colors required to color a map in the plane.

Answer: 4

20. A Hecke algebra is spanned by the double this space of the group it corresponds to, and Gelfand pairs consist of a group and a subgroup such that the subgroup's double these commute. Showing that each of these has the same number of elements is crucial to the proof of ● Lagrange's theorem, which guarantees that the order of a subgroup of a group divides the order of the group. Affine subspaces of a vector space are these, and if a subgroup H is normal, then its set of left these and its set of right these will be equal. For ten points, identify these structures in group theory where a subgroup H is multiplied on either the left or the right by an element g .

Answer: **cosets**