Math Monstrosity, Packet 4

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.
If a question begins with "paper and pencil ready", it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.
If, at any time during an equation, you see something like $\frac{\textbf{THIS}}{2}$ or $\textbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as "this function" or "this quantity" or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say "this thing" or "this".
Pronounciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.
Please read Greek letters as they are (for example, read ϕ as "phi" and not "the golden ratio", even if it represents the golden ratio), with the notable exception of $\sum$ and $\prod$, as in $\sum_{n=1}^{5}$, which should be read as "the sum from $n = 1$ to 5 of".
Similarly, $\int_a^b$ is "the integral from a to b" and $\lim_{n\to\infty}$ is "the limit as n approaches infinity".
In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as "$f$of x" and "lambda of u and v" respectively, and not as "f x" and "lambda u v".
Please read large and/or complex fractions by saying "in the numerator", reading the numerator, saying "in the denominator", reading the denominator, and then saying "end of fraction". For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read "a over b squared plus c".
Please read $\binom{a}{b}$ as "a choose b", not as "a over b".
If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means "the transpose of A", read it as "A to the power of T" or "A superscript T". Most of the notation used in this tournament is common enough that such descriptions, using words like "subscript" and "superscript", should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. This mathematician names the group of Minkowski spacetime isometries, and in dynamical systems, he names the intersection of a periodic orbit with a lower-dimensional subspace also named for him. This mathematician names a homology sphere which results from +1 surgery on the right-handed trefoil knot. This man received a prize from the King of Sweden for his work on the three-body problem, though he did not solve it. This man names a theorem asserting that every simply connected closed ● 3-manifold is homeomorphic to the 3-sphere, the only solved Millenium Problem. For ten points, identify this French mathematician whose namesake conjecture was proven by Grigori Perelman.

Answer: Jules Henri **Poincaré**

2. This quantity is equal to the double integral $\sqrt{EG - F^2}$, where E, F, and G are the coefficients of the first fundamental form. This quantity is equal to the integral of the norm of $\mathbf{T}_u$ cross $\mathbf{T}_v$. The value of this quantity for a zone is equal to $2\pi rh$, and for an oblate ● spheroid it is $2\pi a^2 + \frac{c^2}{e} \ln\left(\frac{1+e}{1-e}\right)$. This quantity is notably infinite for the shape created by revolving the curve $y = \frac{1}{x}$ about the y-axis; that shape is known as Gabriel's Horn. This quantity is the derivative of volume with respect to the inradius of many symmetric shapes. For ten points, identify this property of a three-dimensional shape, which for cubes is equal to 6 times the edge length squared.

Answer: **surface area**

3. Control systems described by this term use various methods of de-thisification including the constraint decision and generalized level set types. Goguen categories are the generalizations of this type of set to category theory. A special case of the sets described by this term are numbers described by this term, which have segmentally continuous membership ● functions that are equal to one at at least one point. If the membership function of an element is equal to one, it is fully included in this type of set, and it is this type of member if its membership function is non-integral. This term is contrasted with crisp. For ten points, identify this term used to describe a type of logic where statements' truth values are taken from the interval $[0, 1]$.

Answer: **fuzzy**

4. The harmonic conjugate to a function f is another function g such that $f + \imath g$ satisfies these equations. Any u and v which satisfy these equations must also satisfy Laplace's equation in two dimensions, and these equations guarantee that any function satisfying them is ● conformal. These equations, which require that the second partial derivatives of u and v with respect to x equal the respective negatives of those derivatives with respect to y, ensure that the derivative is independent of the orientation from which it is taken. Holomorphic functions satisfy, for ten points, which equations which comprise the criterion for a function to be complex differentiable?

Answer: **Cauchy-Riemann** equations

5. A convex one of these for a polytope is defined as the set of vertices of its convex hull, and a cone one of these consists of one point on each edge of a polygonal cone. One of these named for Gröbner is an equivalence system for a set of polynomials. A completely unrelated use of this term is to mean 0.01 percentage points. The existence of one of these named for ● Hamel is justified by the axiom of choice, and allows for the expression of any real number in terms of a certain subset of the real numbers. Despite not actually being one of these, a complete set in a Hilbert space is referred to as a Hilbert one of these. For ten points, identify this term for a set of linearly independent vectors which span a vector space.

Answer: **basis**

6. Babich showed that scrawny examples of these objects could not be defined by tori in a 1992 paper. Any of these sets is, up to homeomorphism, the only totally disconnected perfect compact metric space. One example of these sets also named for Smith and Volterra has positive Lebesgue measure. The most commonly known example of these sets is also known as the ternary set and has fractal dimension less than ● one, and consists of all the real numbers between zero and one whose ternary expansion never contains the digit 1. For ten points, identify this group of nowhere dense perfect sets, one example of which was constructed by iteratively removing the middle third of an interval.

Answer: **Cantor set**s [accept **Cantor comb**, accept **no middle third set** before "middle third"]

7. The Rochester Institute of Technology's hockey arena contains a depiction of these objects in the pavement in front of the entrance. The first of these things to be built was known as the Krämerbrücke, and two of them have since been incorporated into Leninsky prospekt. Three of these things allow individuals to travel to Kniephof, which notably contains the tomb of Immanuel Kant. Two of these things have not been rebuilt since their destruction in the bombing of ● Kalinigrad, the city in which all of these things were located. For ten points, identify these conveyances which allow pedestrians to cross the Pregel River, and were the subject of a seminal problem in graph theory solved by Leonhard Euler.

Answer: Seven **Bridges of Königsberg** [prompt on bridges]

8. Simon Plouffe, in 1998, gave a way to use this process to calculate the binary digits of numbers. The Mohr-Mascheroni theorem states that only one of the two things used for this process is necessary. Pierre Wantzel showed that a conjecture of Gauss pertaining to this process was true; that conjecture stated that this process could generate any ● polygon whose odd prime factors were distinct Fermat primes; notably this means that polygons with 17 and 257 sides can be generated by this process. This process cannot be used to double the cube or trisect the angle. For ten points, identify this ancient Greek method of

geometrical construction which uses two devices, one to draw circles and one to draw lines.

Answer: geometric **construction** by **compass** and **straightedge** [accept synonyms for straightedge like **ruler**, prompt on answers mentioning only construction, or construction and one of the two objects, although accept just construction by compass until "two"; prompt on just compass and straightedge with "what are they being used for?" until "construction" and accept afterward]

9. Strang's strange figures are obtained by plotting functions with this property, and the zeta function with this property is equal to the polygamma function of e to the power of $2\pi\imath x$. A near noble number has a continued fraction expansion with this property. It's not being nowhere differentiable, but the Weierstrass function has this property. A function that can be represented as a generalized Fourier series is said to be almost this. A matrix has this property if some power of it is equal to ● itself, and a function f is said to be Möbius this if it fulfills $f(\theta + \pi) = -f(\theta)$. For ten points, identify this property held by a function if there exists a p such that $f(a + p) = f(a)$ for every a.

Answer: **periodic**ity [accept word forms]

10. The distributive inequalities and the modular identity are true for all of these objects, and these objects are called bounded if their bottom and top are both identities. Heyting and Boolean algebras are special cases of these algebraic objects, an example of which is the set of all partitions of a set with respect to refinement. These structures can be used to represent ● partially ordered sets, and the operations defined on these structures must obey the absorption law and be commutative, associative, and idempotent. For ten points, identify these algebraic structures with operations meet and join, which share a name with a mesh or grid of points.

Answer: **lattice**s [prompt on partially ordered sets or posets until "partially ordered sets"]

11. Cauchy names a theorem which is a partial converse to this statement, and a group for which the converse of this statement is true is called a CLT group; such groups must be solvable and all supersolvable groups are CLT groups. This statement can be used to show Wilson's theorem because it implies that groups of prime order are ● cyclic. One proof of this statement uses the fact that a subgroup's left cosets are equivalence classes. This statement implies that the order of any element of a group divides the order of that group. For ten points, identify this theorem that states that for any subgroup H of a group G, the order of H divides the order of G.

Answer: **Lagrange**'s theorem

12. *Description acceptable.* It has nothing to do with AI, but an article on Nick Bostrom's website entitled "What we should say to the Skeptic" begins by asking "How can we be justified in believing" one answer to this question. In *An Enquiry Concerning Human Understanding*, Hume states that a negative

answer to this question is "no less intelligible" than a positive answer. Pierre-Simon Laplace attempted to answer this question with his rule of succession, but concluded that it was a misapplication of that principle, because the true answer "is far greater for him who, seeing in the totality of phenomena the principle regulating the ● days and seasons, realizes that nothing at present moment can arrest the course of it". For ten points, identify this inquiry about the chances of a certain celestial body's presence in the sky.

Answer: **what is the probability that the Sun will rise tomorrow?** [accept any answer about the **probability** or **likelihood** of **sunrise**; accept answers like "will the Sun rise tomorrow" that don't mention anything related to probability until "Laplace" is read and prompt afterward; prompt throughout on answers like "probability of the Sun exploding" which don't have to do with sunrise]

13. Saccheri proved a number of results about one of these constructs, but believed that his results demonstrated the impossibility of that construct's existence. One of these constructs was modeled by Eugenio Beltrami using the Klein model, and the Cayley-Klein metric was used to describe these constructs. One of these constructs is also known as ● Bolyai-Lobachevskian after the two mathematicians who independently discovered it. The pseudosphere models a section of one of these constructs. Euclid's first 28 propositions are true in all of, for ten points, what constructs formed by negating the parallel postulate, which include hyperbolic and elliptical kinds?

Answer: **non-Euclidean geometry** [prompt on partial answers, prompt on hyperbolic geometry, prompt on elliptical geometry, prompt on geometry, prompt on quadrilateral and accept **non-Euclidean quadrilateral** until "Beltrami", do NOT accept or prompt on "Euclidean geometry"]

14. Propositions 30-32 of Book 7, as well as Proposition 14 of Book 9, of Euclid's Elements constitute a statement of this result. Rings in which this statement holds for ideals are called Dedekind domains. Eisenstein generalized this result to his namesake integers, and Gauss generalized this theorem to the ring of complex integers. This result provides for the uniqueness of ● canonical representations. Rings where this result is true include Euclidean and principal ideal domains and are generally called unique factorization domains. For ten points, identify this result from number theory which guarantees the existence of a unique prime factorization for natural numbers.

Answer: **fundamental theorem of arithmetic** [prompt on fundamental theorem, do not accept or prompt on "fundamental theorem of [anything else]", accept unique prime factorization theorem until "unique prime factorization" is read and prompt afterwards]

15. Steven Wiesner's proposal for unforgeable money unwittingly introduced the idea of these things. These things are operated on by logic gates which send their vectors through unitary transformations. The technique of superdense coding allows these things to possess additional information. Two of these things can

be ● entangled in the Bell state, and information about these things can be gained through standard basis measurement. The Bloch sphere can be used to represent these objects, whose two basis states are ket 0 and ket 1. For ten points, identify these simplest units of information in quantum computers.

Answer: **quantum bits** [accept **qubits**, prompt on bits]

16. A book by J. Scott Carter about this thing describes it as "an Exercise in Diagrammatic Algebra". Rob Kusner proposed an optimal method of doing this based on saddle points in Willmore energy. A recent one of these things was discovered by Aitchison and called the holiverse. One class of these things, their "minimax" type, is constructed by using a half-way model such as Boy's surface or ● Morin's surface. One method for doing this, Thurston's corrugations, was featured in a video produced by the Geometry Center entitled "Outside In". Stephen Smale first identified the possibility of, for ten points, what topological action in which a sphere is turned inside out?

Answer: **sphere eversion** [after "sphere", accept just **eversion**, accept descriptive answers mentioning turning a **sphere** **inside out** until "turned"]

17. Howe's theorem concerns polytopes having this many vertices. Degen's identity guarantees that the product of two sums of this many squares can be written as a sum of this many squares. The "smooth" version of a polygon with this many sides is conjectured to be the least efficiently packable centrally symmetric shape, and any cubic curve passing through this many intersection points of two other cubic curves must pass through all such intersection points. Given any three circles, there are this many circles in the plane to which those three are all ● tangent, and a regular polygon with side length x and this many sides has area $(2 + 2\sqrt{2})x^2$. For ten points, give the number of vertices of a cube.

Answer: 8

18. Modules are like vector spaces except that their coefficients are taken from these and not fields. These entities are called Artinian if they fulfill the descending ● chain condition, and the octonions form a non-associative one of these entities. A commutative one of these entities with a unit element and no divisors of zero is called an integral domain. These entities are referred to as simple if their only two-sided ideals are themselves and zero. Unlike groups, these objects have two binary operations, only one of which requires an identity or inverses. For ten points, identify these algebraic structures with an additive and multiplicative operation, examples of which include the one of integers.

Answer: **ring**s

19. The construction of this set begins by constructing the integers, and then the dyadic rationals, and then the reals (including the non-dyadic rationals). In Von Neumann-Bernays-Gödel set theory, this set is the largest possible ordered field, of which every other ordered field is a subfield. Elements of this set are defined as equivalence classes of forms, which consist of two subsets of this set

enclosed in curly braces and separated by a pipe. It's not the hyperreals, but this set contains positive elements smaller than any ● real number, the simplest of which is denoted by epsilon. For ten points, identify this extension of the real numbers developed by John Horton Conway.

Answer: **surreal** numbers

20. One of these named for Stratonovich acts on semimartingales and processes, and one of these named for Haar is defined for all Borel measurable functions. Khinchin and Denjoy name one of these defined on noncontinuous functions which are the derivatives of continuous functions, and the Henstock-Kurzweil type is used to avoid singularities at a point. A generalization of one of these co-named for ● Stieltjes is defined as the Moore-Smith limit on a directed set of partitions. One of these is defined as the limit of its namesake "sums", which are generated by partitioning the domain of a function. For ten points, identify this type of operation on functions, examples of which are named for Lebesgue and Riemann.

Answer: **integral**s [accept word forms]