Math Monstrosity, Packet 3

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.
If a question begins with "paper and pencil ready", it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.
If, at any time during an equation, you see something like $\frac{\textbf{THIS}}{2}$ or $\textbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as "this function" or "this quantity" or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say "this thing" or "this".
Pronounciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.
Please read Greek letters as they are (for example, read ϕ as "phi" and not "the golden ratio", even if it represents the golden ratio), with the notable exception of $\sum$ and $\prod$, as in $\sum_{n=1}^{5}$, which should be read as "the sum from $n = 1$ to 5 of".
Similarly, $\int_a^b$ is "the integral from a to b" and $\lim_{n\to\infty}$ is "the limit as n approaches infinity".
In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as "$f$of x" and "lambda of u and v" respectively, and not as "f x" and "lambda u v".
Please read large and/or complex fractions by saying "in the numerator", reading the numerator, saying "in the denominator", reading the denominator, and then saying "end of fraction". For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read "a over b squared plus c".
Please read $\binom{a}{b}$ as "a choose b", not as "a over b".
If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means "the transpose of A", read it as "A to the power of T" or "A superscript T". Most of the notation used in this tournament is common enough that such descriptions, using words like "subscript" and "superscript", should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. This man wrote a book entitled "Extraction of the Jewish Era" describing the Jewish calendar. This man's "Book of the Description of the Earth" includes a list of locations by their coordinates. This non-Fibonacci man's second major work, on arithmetic, survives only in its Latin translation, which is commonly referred to as "Dixit [this man]", and introduced ● Arabic numerals to the Western world. This mathematician wrote a book in which he described methods for solving quadratics; the word "algebra" comes from the name of that book. For ten points, identify this Arabic mathematician whose name is the origin of the English word "algorithm".

Answer: Muḥammad ibn Mūsā **al-Khwārizmī**

2. Marden's theorem says that the foci of the Steiner inellipse can be found by solving an equation over this set. Functions from this set to itself can be holomorphic if they fulfill certain conditions. Argand diagrams are used to visually represent elements of this set. On this set, any infinitely differentiable function is ● analytic. The elements of this algebraically closed set can be expressed in phasor form. This set is not an ordered field because it fails to satisfy the requirement that every element's square be positive. For ten points, identify this set of numbers of the form $a + bi$ where $i = \sqrt{-1}$.

Answer: **complex** numbers

3. The number of elements in the free monoid described by this term is given by $\sum_{k=0}^{n} \binom{n}{k} \prod_{i=1}^{k} (k-i+1)^{2^i}$. Numbers are said to be this if they are expressible in the form $\binom{n}{k} k^{n-k}$; this is because the sum from $k = 0$ to n of such numbers gives the number of functions with this property on a set with n elements. Functions having this property include functions mapping sets to their closure and vector space projections. ● Absorbing elements have this property for the binary operations under which they absorb; and identity elements always have this property. For ten points, identify this general property which is held by a member x of a group if $xx = x$.

Answer: **idempotence** [accept word forms]

4. The Segre this is a set of integers describing the orders of the blocks of a Jordan canonical form, while the elliptic this is the first argument in an elliptic integral of the third kind. This term describes a class of functions often described with the Iverson bracket which is defined to be one on a certain set and zero elsewhere, and this term is occasionally used to refer to the floor function of a number. Samuelson's formula allows the computation of the ● polynomial described by this term, which can be written using trace and determinant as coefficients. For ten points, identify this term which describes an equation for a matrix whose solutions are the eigenvalues of that matrix.

Answer: **characteristic**

5. *Description acceptable.* Adelman and Huang developed an errorless version of the elliptic curve one of these. The AKS one of these is the first of them to be polynomial time, but it is practically much slower than the Baillie-PSW one for small inputs. The Miller-Rabin one of these relies on the generalized Riemann hypothesis for its correctness. A simple one of these is named for Fermat and uses his ● Little Theorem, however, for Carmichael numbers it fails to be more efficient than a search for factors. For ten points, identify this class of algorithms to determine if numbers have a certain property, the simplest of which is the Sieve of Eratosthenes.

Answer: **primality tests** [accept any descriptive answer indicating that they are **test**s or **algorithm**s or anything of the like to determine if a number is **prime**, prompt on factoring algorithms]

6. This work opens by stating that "it is a melancholy experience for a professional mathematician to find himself writing about mathematics". This work, which declares that "there is no permanent place in the world for ugly mathematics", also says that "the best mathematics is serious as well as beautiful". This work uses the Euclid's proof of the infinitude of the primes and Pythagoras's proof of the irrationality of $\sqrt{2}$ to provide examples of good mathematics. This work, which argues that pure, or ● "real", math is superior to applied, or "trivial" math, recounts the author's interaction with Srinivasa Ramanujan in which he declared that the number 1729 was boring. For ten points, identify this essay by G. H. Hardy which examines the aesthetics of mathematics.

Answer: A **Mathematician's Apology**

7. Roth's theorem states that if alpha is in this set, then the inequality $\left|\alpha - \frac{p}{q}\right| < \frac{1}{2^{q+\epsilon}}$ has finitely many solutions. The Hermite-Lindemann theorem states that no sum of terms of the form Ae^b, where A and b are members of this set, can equal zero. Conway's constant is a member of this set with ● degree 71. The primitive element of an extension field is always an element of this set. It's not the rationals or the irrationals, but this set is often notated with a double-struck Q with a line over it, to symbolize its role as the "closure" of the rationals. For ten points, identify this set of solutions to polynomials with integer coefficients.

Answer: **algebraic** numbers

8. One approximate method for doing this is found in the Sulba Sutras. This action was mentioned in The Birds by Aristophanes, and Hippocrates attempted to find a way to do this by using lunes. The philosopher Thomas Hobbes believed himself to be capable of doing this, leading to a lengthy debate with John Wallis. The Indiana ● Pi Bill notably contained a proposed method of doing this. This action's impossibility was a corollary of the Lindemann-Weierstrass theorem. For ten points, identify this ancient problem involving the construction of a certain shape with area pi.

Answer: **squaring the circle** [accept descriptive answers like "constructing a square with area pi" or "constructing a square with area the same as a given circle"]

9. The Cesàro one of these objects is also known as the "torn square" one, and the "box" one of these objects is also known as the anticross-stitch curve. Barnsley's Fern and the Minkowski sausage are examples of these objects, another of which, the Gosper ● island, can tile the plane. These objects are associated with a quantity called their "dimension", one kind of which is named for Hausdorff and is usually non-integral. Menger and Koch name two notable examples of, for ten points, which objects which exhibit self-similarity and include the Mandelbrot set and Sierpiński's Triangle?

Answer: **fractal**s

10. *Pencil and paper ready.* This number times π is equal to the area contained by the curve with equation $9x^2 + 16y^2 = 144$. A hexagon with this area would have side length $\frac{2\sqrt{2}}{\sqrt[4]{3}}$. This number is equal to the integral from 1 to m of $x^5 + x$, and it's the area of a cone with radius $2\sqrt{\frac{2}{\pi}}$ and height 9/2. This is the area of the triangle with vertices at the origin, the point $(6, 6)$, and the point $(2, 6)$. ● This is equal to the derivative of $x^2 + 4x$ at $x = 4$, and it's one tenth of 5 factorial. For ten points, identify this number, equal to the perimeter of a square with area 9.

Answer: 12

11. The Gumbel one of these is the first of three types of these things named for Fisher and Tippett. The Erlang one of these simplifies to the exponential one when $h = 1$. One of these things, Student's t-[this thing], was published under a pseudonym by William Gosset. The Bernoulli one of these has two possible outcomes, while the geometric one has a ● probability density function of pq^n. The binomial one of these gives the probability of outcomes from a set of several Bernoulli trials. For ten points, identify this term for functions which describe the probability that certain random data will fall in certain ranges, exemplified by their "normal" type.

Answer: **distribution**s

12. This function satisfies the equation that this function of s is equal to $2^s \cdot \pi^{s-1} \cdot \sin\left(\frac{\pi s}{2}\right) \cdot \Gamma(1-s) \cdot$**THIS**$(1-s)$. This function of 3 is equal to Apery's constant, and it has a closed-form expression for even integers in terms of the Bernoulli numbers. The ● Basel problem asked for the value of this function of 2; that value is $\frac{\pi^2}{6}$. This function of zero is equal to $-\frac{1}{2}$, and this function of -1 is equal to $-\frac{1}{12}$. An important unsolved problem in math asks whether the zeros of this function all lie on the critical line. For ten points, identify this function, which at s is equal to the analytic continuation of the sum of $\frac{1}{n^s}$ for all positive integers n.

Answer: **Riemann zeta function** [prompt on zeta function]

13. A 2004 paper by Cong, Wei, and Zhang describes a thermal-driven method of doing this, and a paper in the same year by Adya and Markov describes the use of Paquet-1 to peform the fixed-outline type of this process. A 1993 paper by

Sarrafzadeh presents a method for converting an arbitrary result of this process into a sliceable one. Constraints on this process include the fact that IP blocks have a predefined area and that bonding pads are usually located on the edges of ● chips. For ten points, identify this process in integrated circuit design where the various circuit components are allocated locations on the chip.

Answer: **floorplanning** [prompt on descriptive answers like "planning integrated circuit layouts]

14. Picard groups are groups of equivalence classes of fractional ideals with this property. If T is a linear operator on a Hilbert space, then the spectrum of T is the set of λ such that $T - \lambda I$ does not have this property on all of the Hilbert space. Maps with this property comprise the elements of the holonomy group of a Riemann manifold, and a square matrix has this property if it has as many ● pivot positions as columns or rows. A knot is said to have this property if it can be transformed into a copy of itself with the orientation reversed. For ten points, identify this property, which is possessed by any matrix which is row-equivalent to the identity matrix and whose eigenvalues are nonzero.

Answer: **invertibility** [accept word forms]

15. Tarski names a theorem stating that these objects form a complete lattice under certain conditions. Unstable and stable stars are two special types of these objects, which also have elliptical and hyperbolic types. Kakutani names a theorem about these which guarantees that a collection of maps with certain properties must share one in common, while ● Schauder names a theorem guaranteeing that some of these exist for a function from a closed convex subset of Banach space to a countably compact subset of that subset. One of these things must exist for any map from a disk to itself. For ten points, identify this term for a point x such that $f(x) = x$.

Answer: **fixed point**s

16. The Seifert Conjecture says that any smooth nonzero one of these structures has at least one closed orbit; it's true for Hopf maps but false in general. The Poincaré-Hopf theorem states that if one of these structures on a manifold fulfills certain conditions, then its index is equal to the Euler characteristic of that manifold. These structures are known as ● central if they are invariant under orthogonal transformation around zero. These structures can be defined generally as sections of the tangent bundle of a differentiable manifold. These structures can be used to represent flow lines. For ten points, identify these structures, operations on which include divergence and curl.

Answer: **vector field**s

17. Feuerbach names one of these shapes passing through the orthocenter of a triangle and centered on the nine-point circle, and Yff names one of these shapes whose vertices are at the centroid and orthocenter of a triangle. These shapes are called rectangular if their ● eccentricity is equal to $\sqrt{2}$, and one of their axes does not actually contain any points on the shape. This shape is defined as the

set of all points with a common difference between their distance from two fixed points. This shape names an alternative set of trigonometric functions based on it rather than the unit circle. For ten points, identify this conic section with two disconnected parts.

Answer: **hyperbola**e [prompt on conic sections before read]

18. *Note to moderators: the bar over a quantity indicates the conjugate of that quantity.* This term describes operators T satisfying $\int_a^b \bar{v}Tu\,dx = \int_a^b uT\bar{v}\,dx$, and the Hilbert-Pólya conjecture states that the nontrivial zeroes of the Riemann zeta function are given by the eigenvalues of an operator described by this term. An inner product described by this term is positive definite and antilinear in the second slot. A "form" described by this adjective is a function fulfilling $f(au + bv, w) = af(u, w) + bf(v, w)$ and ● $f(u, v) = \overline{f(v, u)}$. Every complex matrix can be broken into a part described by this term and a part described by anti-this term. For ten points, identify this eponymous term used to describe self-adjoint matrices, that is, matrices equal to their conjugate transpose.

Answer: **Hermitian**

19. A graph is said to be s-arc-this if an s-route of it exists and there always exists a graph automorphism of it sending each s-route onto every other s-route of it. Two points on a surface have this property if they are opposite each other but not the furthest distance possible. A function is topologically this if for any two intervals A and B, there exists some positive integer k such that iterating f on A k times yields a set disjoint with B. John von Neumann defined ordinal numbers as sets that were ● hereditarily this. A group action has this property if it has only a single group orbit. A relation is referred to as a preorder if it has reflexivity and, for ten points, what property which states that if $x \sim y$ and $y \sim z$, then $x \sim z$?

Answer: **transitive** [accept word forms]

20. Ostrowski's theorem categorizes the different types of this function for the real numbers. The only allowable instance of this function over a finite field is the trivial one, which is equal to zero at zero and is equal to one otherwise. This function must be subadditive, multiplicative, positive-definite, and nonnegative; it is also idempotent and even over the real numbers. The standard metric on the real line is given by this function of the ● difference between two numbers. This function is not complex differentiable anywhere, and its derivative at a nonzero point x in the real line is equal to the sign *[spell out after saying]* of x. For ten points, identify this function which returns x if x is positive and $-x$ if x is negative.

Answer: **absolute value**