

Math Monstrosity, Packet 2

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.

If a question begins with “paper and pencil ready”, it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.

If, at any time during an equation, you see something like $\frac{\mathbf{THIS}}{2}$ or $\mathbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as “this function” or “this quantity” or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say “this thing” or “this”.

Pronunciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.

Please read Greek letters as they are (for example, read ϕ as “phi” and not “the golden ratio”, even if it represents the golden ratio), with the notable exception of \sum and \prod , as in $\sum_{n=1}^5$, which should be read as “the sum from $n = 1$ to 5 of”.

Similarly, \int_a^b is “the integral from a to b ” and $\lim_{n \rightarrow \infty}$ is “the limit as n approaches infinity”.

In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as “ f of x ” and “lambda of u and v ” respectively, and not as “ $f x$ ” and “lambda $u v$ ”.

Please read large and/or complex fractions by saying “in the numerator”, reading the numerator, saying “in the denominator”, reading the denominator, and then saying “end of fraction”. For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read “ a over b squared plus c ”.

Please read $\binom{a}{b}$ as “ a choose b ”, not as “ a over b ”.

If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means “the transpose of A ”, read it as “ A to the power of T ” or “ A superscript T ”. Most of the notation used in this tournament is common enough that such descriptions, using words like “subscript” and “superscript”, should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. This result fails for odd-dimensional surfaces because the alternating sum of their Betti numbers is zero instead of two. As a result of this non-eponymous theorem, any continuous mapping from the surface of a sphere to itself must either have a ● fixed point or a point that maps to its antipode. This theorem implies that some point on the Earth has zero wind velocity, and has been summarized as “you can’t comb a coconut”. For ten points, identify this result stating that every continuous vector field on an even-dimensional spherical surface must have at least one point where the vector is zero.

Answer: hairy ball theorem

2. Tomás Oliveira de Silva has enumerated the free types of these things up to order 28, and Iwan Jensen has enumerated their fixed type up to order 56. The fixed type of these things are also known as animals. One of these things which is self-avoiding and convex, as well as containing two adjacent corners of its bounding rectangle, is known as a “stack” one of these things. Two of these things are compatible if there exists a plane figure which can be tiled by both of them. These things comprise the pieces in the board game ● Blokus, and they are also the primary objects controlled by the player in Tetris. For ten points, identify these things consisting of several connected squares.

Answer: polyominoes

3. Pal Turan names one of these things used to prove the Hardy-Ramanujan theorem, and Viggo Brun names one of these things which was used to prove that the sum of the reciprocals of the twin primes converges. The general number field one of these is able to identify smaller smooth numbers and is therefore more efficient. Sundaram and Atkin each name one of these things which is used to find ● prime numbers, and the quadratic one of these is the fastest factorization algorithm for small integers. For ten points, identify these number theory constructs, the most famous of which identifies prime numbers and is named for Eratosthenes.

Answer: sieves [accept specific sieves]

4. This mathematical construct is featured in a 2005 Foxtrot comic which also features the Perrin sequence. The n th member of this sequence is squareful if (but not only if) n is divisible by 6 or 25. This entity is periodic modulo any m , and those periods are known as ● Pisano periods. The shallow diagonals of Pascal’s triangle sum to members of this sequence, and this sequence’s generating function is $\frac{x}{1-x-x^2}$. The man who names this sequence used it to describe the solution to a problem about breeding rabbits, and Binet’s formula is a closed-form expression for the n th element of this sequence. For ten points, identify this sequence beginning 1, 1, 2, 3.

Answer: Fibonacci sequence

5. A subalgebra of a Lie algebra is called a Cartan subalgebra if it is equal to its normalizer and has this property, and the ring of coquaternions contains

a cone of elements with this property. A Lie algebra is described using this term if its lower central series eventually vanishes, and Lie algebras with this term are diffeomorphic to Euclidean space. This term is used to describe groups whose upper central series terminates with the entire group. This term describes matrices with characteristic polynomial $\bullet x^n$, that is, those whose eigenvalues are all equal to zero. For ten points, identify this property held by an element of a group if some power of it is equal to zero.

Answer: **nilpotent** [accept word forms]

6. This term is used to refer to numbers whose representation on a digital clock, as well as that representation's reflection in both directions and rotation by 180 degrees, is prime. The word "infinite" and this term describe the symmetry group of an apeirogon with two alternating edge lengths. It's not "symmetric", but the smallest \bullet non-Abelian group is the one of order six described by this term, and groups described by this term are expressible as the automorphism group of graphs consisting of only a cycle. Describing the symmetry groups of regular polygons, for ten points, identify this term also used to describe the angle between two planes.

Answer: **dihedral** [accept word forms]

7. A surface is called a this developable of a curve y if it can be parametrized as $\mathbf{x}(u, v) = \mathbf{y}(u) + v\mathbf{A}(u)$, where \mathbf{A} is a vector that also has this property. A square integrable function ϕ has this property if it fulfills the condition $\int \phi(t)^2 dt = 1$. Given a matrix equation $A\mathbf{x} = \mathbf{b}$, the corresponding equation with this property is $\bullet A^T A\mathbf{x} = A^T \mathbf{b}$, and in this equation, $A^T A$ is a matrix with this property. For a subgroup H of a group G , if $xHx^{-1} = H$ for all x in H , then H has this property. A number has this property if every digit and string of digits appears with expected frequency in its decimal expansion. For ten points, identify this property, which is possessed by a vector perpendicular to a plane.

Answer: **normal**

8. The this mapping theorem states that a nonconstant analytic function and a continuous surjective linear mapping between Banach spaces both have this property. Conway and Croft showed that the plane cannot be covered with disjoint congruent circular arcs or line segments with this property. A perforation is a surface that has had a disk with this property removed from it. A function is continuous if its \bullet inverse takes sets with this property to other sets with this property. A set is compact if every cover of this type permits a finite subcover. For ten points, identify this property held by a set A if every element of the set has a neighborhood that is a subset of A , and held by intervals in \mathbb{R} that do not contain their endpoints.

Answer: **openness**

9. One chapter of this book describes a natural language processing computer program and is entitled "SHRDLU, Toy of Man's Designing". This book describes two simple programming languages called BlooP and FlooP in order to

demonstrate the power of unbounded loops. This book describes a phonograph called “Record Player X” which tries to play a record entitled “I Cannot be Played on Record Player X”, invoking one of the title ● figures’ best-known results. Alternating chapters in this work consist of philosophical dialogues involving characters like the Crab, the Tortoise, and Achilles. Subtitled “An Eternal Golden Braid”, for ten points, identify this book by Douglas Hofstadter named for an Austrian logician, a Dutch artist, and a German composer.

Answer: Gödel, Escher, Bach: An Eternal Golden Braid

10. This man’s work in mathematical biology included his paper “The Chemical Basis of Morphogenesis”. This man introduced the method of LU decomposition to solve matrix equations, and also developed a system for the encryption of voice-based communications. His PhD dissertation, Systems of Logic Based on Ordinals, introduced the concept of an oracle machine. He was officially ● pardoned by Queen Elizabeth in 2014, and he developed the Banburismus procedure to aid his work in cryptanalysis. His paper “Computing Machinery and Intelligence” introduced his namesake “test”. For ten points, identify this English mathematician who led the effort to break the German Enigma code.

Answer: Alan Mathison Turing

11. One result with this name is also known as the Fisher-Tippett-Gnedenko theorem, and states that certain limit distributions must belong to one of three families of distributions. This result is used to prove Rolle’s theorem, because in that case it requires either that the function be constant, or that at some point it has derivative zero. This result follows from the statement that closed and bounded sets ● map to closed and bounded sets. For ten points, identify this elementary result from real analysis which says that any continuous function from a closed and bounded set to the real numbers has both a minimum and maximum.

Answer: extreme value theorem

12. Daniell names an axiomatized alternative to this process. Fatou’s lemma relates this operation on the limit inferior of a sequence of functions to the limit inferior of the sequence of this operation of those functions. A theorem states that if a sequence of functions f_n is all less than a function g on which this process can be performed, then this process can be performed on the limit of that sequence; that is the ● dominated convergence theorem. This process is named for a mathematician who also names the measure on which it is based. For ten points, identify this more powerful generalization of the Riemann integral.

Answer: Lebesgue integral [accept word forms, prompt on integral and word forms, do not accept or prompt on any other sort of integral like “Darboux integral”]

13. An article in a September 1992 issue of Computer Shopper detailed how this programming language took 17 hours to complete a benchmark that C performed in half a second. This programming language’s manual contains a tonsil,

because “all other reference manuals have Appendices, [so] it was decided that [this language’s] manual should contain some other type of removable organ”. This language uses a question mark for XOR to “correctly express the average person’s reaction on first encountering exclusive-or”. Programs in this language will be rejected for improper quantities of ● politeness unless an appropriate fraction of lines begin with the word PLEASE. For ten points, identify this esoteric programming language, an abbreviation of “The Compiler Language with No Pronounceable Acronym”.

Answer: INTERCAL [accept The Compiler Language with No Pronounceable Acronym until read]

14. These things are generalized by CW-complexes. The Kuratowski reduction theorem states that any of these things with a certain property must have one of two specific ones of these things as a minor. They’re not matrices, but these things have spectra consisting of the set of their eigenvalues. Products on these things include the strong, categorical, and lexicographic products. If these things are disconnected, then they have infinite ● diameter. One of these things is called a tournament if it is complete and oriented. The bipartite type of these things is defined by having a chromatic number of two. For ten points, identify these mathematical objects consisting of a set of points connected by edges.

Answer: graphs

15. A mathematician from this country names a special case of the principle of least action stating that physical systems follow the path with the shortest length. A mathematician from this country names lattices generated by discrete translation operators. Another mathematician from this country introduced the adèle ring. The equation $ax^2 + by^2 + cz^2 = 0$ is named for a mathematician from this country, and a different mathematician from this country names the standard ● measure used for Euclidean space. A mathematician from this country names series used to represent functions as the sums of sine waves. For ten points, identify this country, home to Bravais, Fourier, and Lagrange.

Answer: France [liberally accept historical names like “Third French Republic”]

16. Madhava names a special case of these entities, and Laurent names an extension of these entities which can be used to accommodate discontinuities. If a function is equal to one of these for all complex numbers it is known as ● entire. Notably, one of these for the function $e^{-\frac{1}{x^2}}$ is identically zero because all of the function’s derivatives are zero at zero. Lagrange names a formula for the error present in one of these entities, also known as the remainder. A special case of these entities is named for Maclaurin, that is when these are centered at zero. For ten points, identify these approximations of functions through power series.

Answer: Taylor series [antiprompt on Maclaurin series until “Maclaurin”, prompt on power series until read]

17. Eight of the eleven rings with the same number of elements as this group have its operation as their additive operation, and all of this group's 5 subgroups are normal. This group names an a cappella singing group at Northwestern University best known for their song "Finite Simple Group of Order Two". This group is isomorphic to $\bullet D_2$, the second dihedral group, and to $Z_2 \times Z_2$. This group is equal to the symmetry group of a rectangle which is not a square. This group is the smallest non-cyclic group, and every element's square is equal to the identity. For ten points, identify this group with four elements named after a mathematician who also names a bottle.

Answer: **Klein four-group** [prompt on four-group, accept vierergruppe, accept D_2 and $Z_2 \times Z_2$ before read]

18. Tarski constructed an elegant second-order axiomatization of this structure. Adding two improper elements to this set gives its affinely extended version, while adding one gives its projectively extended version, which can be represented by a cross-section of the Riemannian sphere. This set is the unique \bullet complete Archimedean field, as well as the unique Dedekind-complete ordered field. This set's elements can be expressed as the limits of sequences of rational numbers. For ten points, identify this set of numbers represented with a double-struck \mathbb{R} .

Answer: **real** numbers

19. According to Donald Knuth, the first study of these objects was performed by Bhāskarāchārya, and alternate names for them included "bunch" and "suite". Replacing the falling factorial with a rising factorial in the expression for binomial coefficients gives the number of these of a given size with a certain number of elements. The free commutative monoid on a set can be interpreted as the monoid of these things with elements drawn from that set. These things are formally defined using a set and a function from that set to the \bullet positive integers. For ten points, identify these objects which generalize sets to allow repeated instances of elements.

Answer: **multisets** [be generous and prompt on sets, also prompt on bags, lists, heaps, samples, weighted sets, and collections, prompt on bunches and suites until they are respectively read]

20. The number of this kind of degree- n polynomials mod 2 is equal to the number of binary Lyndon words with length n . Any representation of a finite Lie group can be expressed as a direct sum of this kind of representations, and the number of this kind of representations of a group is equal to the number of conjugacy classes of the group. This term is used to describe ring ideals not expressible as the \bullet intersection of two other ideals. $x^2 + 1$ is this over the integers, but is not this mod 2 because it is the product of $x + 1$ with itself. For ten points, identify this term used to describe polynomials that cannot be factored.

Answer: **irreducible** [accept word forms]