Math Monstrosity, Packet 12

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.

If a question begins with "paper and pencil ready", it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.

If, at any time during an equation, you see something like $\frac{\textbf{THIS}}{2}$ or $\textbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as "this function" or "this quantity" or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say "this thing" or "this".

Pronounciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.

Please read Greek letters as they are (for example, read ϕ as "phi" and not "the golden ratio", even if it represents the golden ratio), with the notable exception of $\sum$ and $\prod$, as in $\sum_{n=1}^{5}$, which should be read as "the sum from $n=1$ to 5 of".

Similarly, $\int_a^b$ is "the integral from a to b" and $\lim_{n\to\infty}$ is "the limit as n approaches infinity".

In general, something of the form $f(x)$ or $\lambda(u,v)$ is a function, and should be read as "f of x" and "lambda of u and v" respectively, and not as "f x" and "lambda u v".

Please read large and/or complex fractions by saying "in the numerator", reading the numerator, saying "in the denominator", reading the denominator, and then saying "end of fraction". For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read "a over b squared plus c".

Please read $\binom{a}{b}$ as "a choose b", not as "a over b".

If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means "the transpose of A", read it as "A to the power of T" or "A superscript T". Most of the notation used in this tournament is common enough that such descriptions, using words like "subscript" and "superscript", should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. A branch of set theory named after this thing was developed by Howard Hanson; in that field, sets which can be transformed into one another by inversion or transposition are said to belong to the same set class. David Lewin developed transformational theory, a way to apply groups to the theory of this thing, in his paper "Generalized [This Thing] Intervals and Transformations". A book by Dave Benson describes this thing as "a Mathematical ● Offering", and has chapters entitled "Consonance and dissonance" and "Scales and temperaments: the fivefold way". For ten points, identify this fine art practiced by people such as Philip Glass and Pyotr Chaikovskiy.

Answer: **music**

2. The Lebesgue differentiation theorem is a generalization of this statement, although in order for the converse to be satisfied one must use Henstock-Kurzweil integration. The time evolution of integrals generalizes this statement in specific contexts, and the gradient and divergence theorems generalize this statement. ● Stokes's theorem is the multidimensional generalization of this statement which formally establishes that antidifferentiation and integration are the same thing. For ten points, identify this central result which states that the derivative of the integral of a function is that function.

Answer: **fundamental theorem of calculus** [prompt on fundamental theorem, do not accept "fundamental theorem of [anything else]"]

3. S. George Pemberton authored a paper titled after the biogenic version of this property, which is attained through bioturbation of sediment. A paper by Berryman and Millsten argues that ecosystems do not normally have this property, but that human intervention could cause them to have it by exacerbating feedback loops. Systems with this property must exhibit topological mixing and dense periodic orbits. The ● logistic map has this property, as does a system consisting of a magnetic pendulum moving around three fixed magnets. A simple system with this property is the double-rod pendulum, and a commonly used example is the Lorenz attractor. For ten points, identify this property held by a system susceptible to small changes in initial conditions through the so-called "butterfly effect".

Answer: **chaos** [accept word forms]

4. The criss-cross and ellipsoid algorithms are methods for doing this process, for which Khachiyan developed the first polynomial-time algorithm. Karmakar's algorithm, a highly efficient algorithm for this process, is an interior point method. By adding slack variables, equations in this process can be converted into augmented form, allowing the application of the simplex algorithm, which operates by moving between the vertices of the ● feasible region until it finds the solution. For ten points, identify this process by which a linear function of several variables is optimized over a region constrained by several inequalities.

Answer: **linear programming**

5. Frattini names one of these represented by phi which is the intersection of all of the maximal ones of these. These objects have a "Fitting" type generated by others of these objects which fulfill certain conditions. The Jordan-Hölder theorem guarantees pairwise isomorphism between sequences of these objects called composition series. The socle is one of these objects, and one of these objects is said to be ● normal if it is invariant under the similarity transformation for all x. The local type of these objects acts as a normalizer for nontrivial Sylow p-these. For ten points, identify these entities in abstract algebra consisting of some of the elements of a group but still fulfilling all of the group axioms.

Answer: **subgroup**s [prompt on groups]

6. This technique uses the Löwenheim-Skolem theorem and the Mostowski collapsing theorem to ensure that objects in it are countable and transitive. This technique uses its namesake posets, which satisfy a splitting condition and are associated with a class of names. The original version of this technique is now called its ramified version, which is distinguished from the newer unramified version. This technique's first major application was in proving the independence of ● Zermelo-Fraenkel set theory from the axiom of choice and from the continuum hypothesis. For ten points, identify this technique of proof used in set theory and developed by Paul Cohen.

Answer: **forcing**

7. Every sequence of integers possessing a positive value of the upper Banach version of this quantity must contain arbitrarily long arithmetic progressions by Szemerédi's theorem. Mann's theorem states that for two sets of integers A and B, sigma of the direct sum of A and B is greater than or equal to $\min\left(1, \sigma\left(A\right) + \sigma\left(B\right)\right)$, where sigma is the Schnirelmann version of this quantity. The smoothed octagon is conjectured to have the lowest value of the ● packing version of this quantity in two dimensions, while the circle has the highest such value for this quantity. Integrating this quantity's function of a lamina gives its mass. For ten points, identify this quantity, which in physics could be measured in units of kilograms per liter.

Answer: **density**

8. Kochanek and Bartels name one of these which is equipped with continuity, tension, and bias parameters, and the centripetal Catmull-Rom one lacks cusps or self-intersections. The Irwin-Hall distribution is an example of one of these, and one of these of degree m is called perfect if its mth derivative is equal to plus or minus 1. The cubic Hermite type of these are defined by their values and first derivatives at a certain set of points, and the non-uniform rational B type of these is commonly used in ● computer graphics. Bézier curves are an example of, for ten points, what kind of object which consists of smoothly connected piecewise polynomial functions?

Answer: **spline**s

9. A book by Guerino Mazzola connects music to this field of mathematics. Many practitioners of this field refer to breaking equivalence-invariance as

"evil". A subset of this field of mathematics concerns a generalization of point-set topologies which have elementary and Grothendieck types. A fundamental result in this branch of mathematics generalizes Cayley's theorem and makes statements about ● functors which are morphisms onto a fixed object; that result is the Yoneda lemma. Topos theory is a subset of, for ten points, what this field of mathematics which deals with collections of objects and arrows, the so-called "abstract nonsense"?

Answer: **category theory** [accept **abstract nonsense** before "abstract", accept **topos theory** before "topos"]

10. *Pencil and paper ready.* This number is equal to the magnitude of the gradient of $xy + yz$ at the point $(1, 4, 1)$. The trigonal trapezohedron has this many faces, and the volume of a tetrahedron with side length x is $\frac{x^3}{\textbf{THIS} \cdot \sqrt{2}}$. The integral from 2 to 5 of x^2 is three more than the square of this number. This number is equal to the dot product of the vectors $\langle 5, 2 \rangle$ and $\langle 4, -7 \rangle$. ● The geometric mean of 24 and this number is 12, and the sine of π over this number is $\frac{1}{2}$. For ten points, identify this number equal to the number of vertices of an octahedron and the cube root of 216.

Answer: 6

11. This man's barrel is an experiment used to demonstrate concepts of hydrostatics. This man wrote a work in which he argued for formalism in definition and eschewed intuition. This inventor of the syringe argued that the axiomatic method necessitates submission to God. This man collaborated with the Chevalier de Méré to solve a problem about ● dice games, which led to the establishment of the mathematical theory of probability. This man had an intense religious experience which led him to give up mathematics, and is known for formulating an argument for religion based on expected value. For ten points, identify this French mathematician who names both a wager and a triangle.

Answer: Blaise **Pascal**

12. There are two definitions of this term for polynomials, the first being the smallest k such that the polynomial divides $x^k + 1$, and the second being the same as the degree of the polynomial. A group is a p'-group if p does not divide this quantity for the group. A group involution is an element of a group such that this quantity is equal to ● two for that element, and if this quantity for a group is prime there is only one possible group with that value for this quantity, and it must be cyclic. For ten points, identify this term used to refer to the smallest k such that $g^k = 1$ for an element g of a group, and also used to refer to the number of elements of a group.

Answer: **order**

13. Hendrickson and Kolda described a method to use graph partitioning models to describe the execution of this process, while a 1994 book by Michael Quinn is entitled *[this thing]: theory and practice*. Erlang and Occam are programming languages which can be used for this. The extent to which an algorithm can

be sped up by using this technique is described by ● Amdahl's law. This technique is split into bit-level, instruction-level, and task types. Examples of hardware which executes this paradigm are clusters and multi-core processors. For ten points, identify this computing paradigm in which multiple operations are simultaneously executed by separate units.

Answer: **parallel computing** [accept just **parallelization** and word forms, accept specific types of it like **distributed computing** unless they've already been read (in which case prompt), accept **concurrent computing** because they're frequently conflated]

14. One holder of this position wrote *The Elements of Algebra in Ten Books* and was notably blind. Another holder of this position discovered, with Schwarz, that the anomaly in type I string theory cancels. A holder of this position names a theorem relating the surface integral of the curl of a vector field to the line integral over its boundary. A holder of this position developed a "difference engine", while another names a ● delta function. The second holder of this position names three laws of motion. *A Brief History of Time* was written by, for ten points, a holder of what Cambridge professorship held by Charles Babbage, Isaac Newton, and Stephen Hawking?

Answer: **Lucasian Chair** of Mathematics [accept "Professor" for "Chair" and word forms thereof]

15. The trace type of these entities is used as a model for concurrent computation, and the history type of these entities arises in the same field. Semiautomata are associated with one of these entities called their characteristic or transition type. For a formal language L, the syntactic one of these objects is the smallest that recognizes L. The Kleene star is used to define the construction of the free type of these entities. The positive integers form one of these constructs under ● multiplication, and these constructs are equivalent to semigroups endowed with an identity. For ten points, identify this mathematical structure with an associative binary operation and identity element.

Answer: **monoid**s

16. A generalization of this concept is used to differentiate time-dependent functions of stochastic properties; that generalization is called Itô's lemma and generalizes this concept to the stochastic calculus. The triple product rule is also known as Euler's this. The multidimensional analog of this identity can be used to show that the Jacobian of the ● composition of two functions is the product of the Jacobians of the two functions, when evaluated at the appropriate points. For ten points, identify this method of differentiating the composition of two functions.

Answer: **chain rule**

17. The possible ones of these on some set form a lattice ordered by set inclusion called Con, and if a set is accompanied by one of these then it is referred to as a setoid. Equality is the finest one of these on any set, and if a construct

is Euclidean and reflexive then it is one of these. This kind of construct is a symmetric preorder and a transitive tolerance relation, and these objects ● partition a set into their namesake classes. Examples of this type of relation include geometric similarity and modular congruence. For ten points, identify this kind of relation which must be reflexive, symmetric, and transitive.

Answer: **equivalence** relation [do not accept or prompt on "equality"]

18. Taylor's condition requires that the number of vertices n of a weighted tree be either one of these or two more than one of these numbers in order for its path lengths to be distinct and consist of the set of integers between 1 and n choose 2 inclusive. The generating function for these numbers is $\frac{x(x+1)}{1-x^3}$. Fermat's sandwich theorem states that only one positive integer is sandwiched between one of these numbers and a perfect ● cube, and only 31 numbers cannot be expressed as the sum of distinct ones of these. Every positive integer can be expressed as the sum of no more than four of these numbers, and these numbers cannot end in 2, 3, 7, or 8 mod 10. For ten points, identify these products of an integer with itself.

Answer: perfect **square** numbers

19. A philosophical dialogue by this man begins with a man praying to God to absolve him of free will and is called "Is God a Taoist?" One of this man's books provides the reader with an introduction to combinatory logic in which combinators are depicted as birds that repeat songs; that book is *To Mock a Mockingbird*. This man's *Forever Undecided* contains ● conundrums centered around undecidability, and his *Satan, Cantor, and Infinity* focuses on set-theoretic concepts of infinity. For ten points, identify this logician, many of whose works, including *What is the Name of This Book?*, are collections of logic puzzles.

Answer: Raymond Merrill **Smullyan**

20. An axiom named for this property must be added to Euclidean geometry to guarantee that certain circles intersect each other. A function has this property at all points where its oscillation is zero, and places where functions lack this property can be divided into infinite and ● removable types. A function has this property if the limit of the function at every point is equal to the actual value of the function there, and functions with this property are said to be C^0-type. This property is notably not held by the reciprocal function because of its vertical asymptote. For ten points, identify this property held by functions whose graphs can be drawn without picking up one's pencil.

Answer: **continuity**