Math Monstrosity, Packet 1

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1 General Instructions to Moderators

1.1 For everyone: question formatting specific to this tournament

Power is denoted by a black circle, ●. Buzzes before the circle should be awarded power. The question is not bolded before the powermark, so please make sure you're awarding power correctly.
If a question begins with "paper and pencil ready", it is a computation question. Please read such questions slowly and pause for 2-3 seconds between clues.
If, at any time during an equation, you see something like $\frac{\textbf{THIS}}{2}$ or $\textbf{THIS}(n)$, then the word **THIS** refers to the thing being asked for in the question. If you're comfortable enough with math that you know what's going on, please read that as "this function" or "this quantity" or whatnot. If you're not, you can either parrot pronouns used earlier in the tossup, or just say "this thing" or "this".
Pronounciation guides are *[in brackets and italics]*.

1.2 For people who don't know how to read math: how to read math

In general, spell acronyms out. I will make sure to include a reading guide if this is not the case.
Please read Greek letters as they are (for example, read ϕ as "phi" and not "the golden ratio", even if it represents the golden ratio), with the notable exception of $\sum$ and $\prod$, as in $\sum_{n=1}^{5}$, which should be read as "the sum from $n = 1$ to 5 of".
Similarly, $\int_a^b$ is "the integral from a to b" and $\lim_{n\to\infty}$ is "the limit as n approaches infinity".
In general, something of the form $f(x)$ or $\lambda(u, v)$ is a function, and should be read as "$f$of x" and "lambda of u and v" respectively, and not as "f x" and "lambda u v".
Please read large and/or complex fractions by saying "in the numerator", reading the numerator, saying "in the denominator", reading the denominator, and then saying "end of fraction". For simpler fractions, like $\frac{a}{b^2+c}$, you can simply read "a over b squared plus c".
Please read $\binom{a}{b}$ as "a choose b", not as "a over b".
If you are not familiar with a certain piece of mathematical notation, please do your best to describe it to the players; for example, if you don't know that A^T means "the transpose of A", read it as "A to the power of T" or "A superscript T". Most of the notation used in this tournament is common enough that such descriptions, using words like "subscript" and "superscript", should suffice. If there are any problems which use particularly arcane notation, I will make sure to provide a reading guide.

2 Tossups

1. *Pencil and paper ready.* This number times π is the volume of a solid formed by rotating the area between the curve $y = \frac{1}{x}$, the y-axis and the lines $y = 1$ and $y = 2$ about the y-axis. This number is equal to $\int_1^\infty \frac{1}{x^3}\,dx$. This number times 5 is equal to the derivative at $x = 1$ of $x^2 + \sqrt{x}$. This number is equal to the determinant of the inverse matrix of $\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$. ● The inverse hyperbolic cosine of twice this number is zero, and $\sin \frac{\pi}{6}$ is equal to this number. For ten points, identify this number, equal to the probability of rolling an odd number on a single regular die.

Answer: $\frac{1}{2}$

2. This statement can be proved by using the fact that the game of hex cannot end in a draw. This statement, which is equivalent to the Knaster-Kuratowski-Mazurkiewicz lemma, has a generalized version named for Schauder. This result is used in the proof of the Borsk-Ulam theorem. This result stemmed from a novel proof of the ● hairy ball theorem using homotopy, and its first proof relied on the notion of degree. This statement is equivalent to a statement about labeling the edges of triangles called Sperner's Lemma. For ten points, identify this theorem stating that any continuous map from a closed ball to itself must map at least one point to itself.

Answer: **Brouwer fixed point theorem** [prompt on fixed point theorem]

3. The Whitney sum is also known as the this bundle. When performing this operation, the output must have a finite number of nonzero elements. This operation comes equipped with a projection and coprojection homomorphism. This operation is a biproduct when it is performed over a finite number of modules, and it is always a coproduct in the appropriate category for the objects on which it asks. For a finite number of sets, this operation is equal to the ● Cartesian product of those sets, and for an infinite number of Abelian groups, this operation on the groups is a proper subset of their direct product. For ten points, identify this algebraic operation denoted by an encircled plus sign.

Answer: **direct sum** [prompt on sum, prompt on addition]

4. One mathematician from this country names an equation which states that $AXA^H - X + Q = 0$, where Q is a Hermitian matrix. Another mathematician from this country names the fact that there are more primes of form $4k + 3$ than $4k + 1$; that is his bias. A mathematician born in this non-Germany country developed the ● transfinite numbers and the idea of 1-to-1 correspondences. Georg Cantor was born in this country, and while it's not Switzerland, Leonhard Euler spent much of his adult life and died in this country. A mathematician from this country names the length of the shortest computer program that produces some object; that is his complexity. For ten points, identify this country, home to Pafnuty Chebyshev and Sofia Kovalevskaya.

Answer: **Russia**n Federation [accept **Rossiya**, liberally accept historical names like "Soviet Union" or "Russian Empire", liberally accept those same names in Russian]

5. Barban's constant and Murata's constant are defined as products over these numbers. Pépin's theorem gives conditions for Fermat numbers to hold this property. If the radix of a numeral system has this property, then there are an infinite number of integers with this property ending with each nonzero digit. Bertrand's postulate, which was proved by Chebyshev and later by ● Erdős, states that for any integer n there is always at least one number with this property between n and $2n$. These numbers are all congruent to 1 or 5 mod 6. For ten points, identify these numbers which have two distinct postive divisors.

Answer: **prime** numbers [accept word forms like **primality** for the property]

6. Casper Goffman's 1969 article referencing this quantity in its title is believed to be the first printed mention of this quantity. Tompa proposed a directed graph version of this quantity, and Michael Barr suggested a rational version. Oakland University maintains a comprehensive online database dedicated to this quantity, including a list of ● people who possess low values of it. The earliest known person to have a finite value of this quantity is Pierre-Simon Laplace, for whom it is 14; notably, all Fields Medalists have a finite value of this quantity that is less than or equal to six. For ten points, identify this quantity representing one's collaboration distance to a certain Hungarian mathematician.

Answer: **Erdős number** [prompt on collaboration distance or academic distance]

7. The colon one of these acts on dyads, and the Pippenger one of these is a formula for e. Hadamard names one of these which represents the Riemann zeta function in terms of its nontrivial zeros, and one of these entities named for Jordan is defined as $\frac{1}{2}(xy + yx)$. The cup one of these, which acts on cohomology classes, can be represented by the ● wedge one of these if it is acting on closed forms. The Kronecker one of these is also known as the matrix direct one, and the scalar triple one of these can be used to compute the volume of a parallelepiped. For ten points, identify this general term used to describe a number of binary operations, which include Cartesian, dot, and cross kinds.

Answer: **product**s [do not accept or prompt on "multiplication"]

8. This non-American mathematician names a theorem stating that a 1-to-1 correspondence exists between division rings of finite index and unital algebras of finite dimension. This mathematician also co-names a theorem stating that progressive functions have fixed points on posets. This mathematician introduced the dangerous bend symbol as well as the symbol for the empty set, and also coined the terms injective, surjective, and bijective. This mathematican's main work, ● *Elements of Mathematics*, axiomatically constructs the core of modern mathematics. For ten points, identify this non-existent mathematician, the shared pseudonym of a collective of mainly French mathematicians.

Answer: Nicolas **Bourbaki**

9. Allen Yang developed a means of using this technique to recognize obscured and corrupted faces. Fornasier and Rauhut describe, in a paper named after this technique, how algorithms such as ℓ_1-minimization could be used for it. This technique has enabled the development of single-pixel cameras that take a series of measurements and use this technique to analyze them. A paper by Mark Davenport et al entitled "Introduction to [this]" describes how this technique could surpass bounds set by the ● Nyquist-Shannon sampling theorem. For ten points, identify this technique in signal processing which allows for sparse signals to be recovered from far fewer samples than would normally be possible.

Answer: **compressed sensing**

10. One of these things is equal to the graded Euler characteristic of a Floer homology of Ostváth and Szabó. A paper by Edward Witten entitled "Quantum Field Theory" and [one of these things] showed that two of these things have correspondences to Chern-Simons gauge theory. One of these things is in two variables and is named for its six discoverers; that one of these things is the ● HOMFLY *[read as: hom-flea]* one. The first-discovered one of these things is symmetric and named for Alexander. Another of these is defined through a skein relation and takes the value of 1 on the unknot; that one is named for Jones. For ten points, identify these algebraic objects which are used as invariants to describe knots.

Answer: **knot polynomial**s [after "knots" is read, accept just "polynomials"]

11. This function satisfies the equation $\liminf_{n\to\infty} \mathbf{THIS}(n)\frac{\ln\ln n}{n} = e^{-\gamma}$. Lehmer names a problem asking whether there exists an n such that this function of n divides $n-1$; no such numbers are known. A theorem named for this function states that for all a relatively prime to n, $a^{\mathbf{THIS}(n)}$ is congruent to ● 1 mod n. For prime numbers p, this function of p is equal to $p-1$. For ten points, identify this function which gives the number of integers less than a certain integer which are relatively prime to it and is symbolized by a phi.

Answer: Euler's **totient** function [prompt on Euler's phi function until read]

12. This thing is a special case of a similar thing named for Gelfand, and can be generalized to a discrete-time version over the integers. Plancherel's formula, which allows this thing to be extended to a unitary operator, relates the integral of the square of a function to the integral of the square of this thing of that function. This thing has the properties of translation, modulation, and time scaling. This thing's ● fast type is an algorithm for computing its discrete type. For ten points, identify this means of expressing a function as a sum of trigonometric functions.

Answer: **Fourier transform** [prompt on anything with Fourier in it like "Fourier series"]

13. One of these things comprises the coat of arms of the Statistical Society of Canada. A stained-glass window in Caius College, Cambridge depicts one of these and honors Ronald Fisher, whose work *Design of Experiments* used

these things for experimental design. McKay and Rogoyski, in a 1995 paper, counted the number of these things with order 10 and higher. These things can be divided into isotopy classes and expressed in orthogonal array representation. Solutions to ● Kenken and Sudoku puzzles are examples of these things. For ten points, identify these $n \times n$ arrays which contain every integer from 1 to n exactly once in each row and column.

Answer: **Latin square**s [prompt on Sudoku solutions and Kenken solutions until they are respectively read]

14. Kyrmse proposed the term "polynema" to describe graphs which have this property and some fixed number of edges. The topologist's sine curve has this property, but is not locally this. A space is said to be hyper-this if no two open sets are ● disjoint. A space has the opposite of this property if it can be expressed as the union of two disjoint open sets. The real line has this property, but if any point is removed, it loses this property. For ten points, identify this property, whose path version is possessed by a space if there exists a path between any two points in the space lying entirely within the space.

Answer: **connected**ness [accept word forms]

15. This shape's evolute is referred to as the "semicubical" version of it. A section of this shape is a Lissajous curve with $\omega = 2$ and $\delta = \frac{\pi}{2}$. The equation $p^2 = ar$ represents this shape in pedal coordinates. When this shape is inverted with the inversion center at its vertex, the result is the cissoid of Diocles. Menaechmus successfully duplicated the ● cube using two of these shapes. An equation for this curve in polar coordinates is $r = -\frac{2a}{1+\cos\theta}$, and a mirror with this shape focuses parallel rays of light to one point. For ten points, identify this shape defined as the set of all points equidistant from the focus and the directrix, one of which is described by the graph of the equation $y = x^2$.

Answer: **parabola**

16. The Sol geometry is the geometry of the semidirect product of one of these structures and $\mathbb{R}^2$. The Baker-Campbell-Hausdorff theorem connects these objects to algebras described by the same term, and Heisenberg and Lorenz name examples of these. A homomorphism between two complex instances of these objects must be a holomorphic map, and the regular Fréchet kind of these are ● diffeomorphism groups of compact manifolds. This groups are defined by the requirement that the mapping $(x, y) \to x^{-1}y$ is smooth from the product manifold into the group. For ten points, identify this class of groups which are also differentiable manifolds.

Answer: **Lie** groups [do not accept "Lie algebras"]

17. The Krull-Schmidt theorem is an important result in this branch of mathematics which also includes the Mackey formula. In the modular type of this branch of mathematics, Maschke's theorem does not hold because a certain entity is not invertible. If an equivariant map exists between two vector spaces in this field of study, they are isomorphic examples of the namesake object of this

field of study, which is a homomorphism from a group or algebra to a set of ● matrices. For ten points, identify this field of mathematics concerned with expressing the elements of algebraic structures as linear transformations of vector spaces.

Answer: **representation theory**

18. Only sets with this property can be subselfsimilar, and a subset of the reals is said to be an F_σ set if it is the countable union of sets with this property. A compact manifold has this property if it has no boundary, and if a set is bounded and has this property, then its Minkowski measure is the same as its Lebesgue measure. The singular ● support of a function is a set with this property where the function is not smooth. A curve that has this property and is simple is called a Jordan curve. The union of a set and its accumulation points always has this property, just like any shape that contains its own boundary. For ten points, identify this property which is held by sets whose complements are open.

Answer: **closure** [accept word forms like **closed**]

19. The Hahn-Mazurkiewicz theorem states that these things can only exist in spaces fulfilling certain criteria. These things, which can never be differentiable, include one named for Osgood which is non-self-intersecting and one named for Gosper which is also known as the ● "flowsnake" and covers the Gosper island. One of these things was constructed in order to create a surjective mapping from the unit interval to the unit square; that one is named for Peano, while another one with the same property is named for Hilbert. For ten points, identify this class of plane curves which completely cover a given multidimensional area.

Answer: **space-filling curves** [accept **Peano curves** until "Peano" is read, accept **plane-filling curves**]

20. Smoluchowski names an example of the stochastic type of these, another example of which are named for Langevin. Picard and Lindelöf name a theorem giving conditions under which these things are solvable. The shallow water and diffusion ones are examples of these used in physics, and these things are called ● homogenous if any linear combination of some of their solutions is another solution. Maxwell names a set of these which feature prominently in electrodynamics. These entities come in ordinary, linear, and partial types, among others. For ten points, identify this kind of equation which relates a function to one or more of its derivatives.

Answer: **differential equations** [accept specific types of differential equations]